Volatility versus Tail Risk: *Which One Is Compensated in Equity Funds?*

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We are motivated to test whether volatility and tail risk are both compensated in the equity fund universe. We address both issues at the same time so that our study can directly answer the question: If volatility is not compensated, what about tail risk? We choose equity funds instead of individual stocks, because systematic risk is more relevant to investors' portfolios. In a typical portfolio setting, idiosyncratic risk is largely diversified away.

Extensive empirical evidences show that low-volatility or low-beta portfolios historically have offered higher realized average stock returns than comparable high-volatility or high-beta portfolios constructed from the same starting universe of individual securities. Ang et al. [2006, 2009] found that high-risk stocks have had abysmally low average returns in longer U.S. samples and in international markets, rekindling interest to the low-volatility anomaly. Blitz and van Vliet [2007] demonstrated its robustness across regions. Frazzini and Pedersen [2010] documented the low-volatility anomaly in global stock, Treasury, credit, and futures markets. Finally, Baker et al. [2011] provided an excellent review on the low-volatility and low-beta anomalies.

This historical finding is counter to the fundamental principle that higher risk is compensated with higher expected return. Several behavioral models, including a preference for lotteries and arbitrage limits, have been proposed to explain this anomaly. For example, Baker et al. [2011] suggest that the typical institutional investor's mandate to beat a fixed benchmark, such as the S&P 500 index, could increase the demand for high-beta stocks and thus lower expected returns. Hsu et al. [2013] provide an alternative explanation for the low-volatility puzzle, hypothesizing that analysts inflate earnings forecasts more aggressively for volatile stocks. Because investors overreact to analyst forecasts, this can lead to systematic overvaluation and low returns for high-volatility stocks.

We ask a different question: If volatility or beta is not compensated within a particular universe of similar securities, are there other types of risk that might be compensated? More specifically, in contrast to risk measures, such as volatility or beta, that penalize upside gains just as they penalize downside losses and fail to account for non-normal return characteristics, perhaps a type of risk, such as tail risk, that investors unambiguously view as bad might garner a tail-risk premium. Volatility or beta may not be a relevant risk measure, but tail risk might be a candidate for higher compensation.

A large negative event can significantly reduce portfolio value. Examples include the 1929 stock market crash, 1987's Black

Monday, the 1997 Asia Crisis, the 2000 dot-com bubble burst, and the 2008 financial crisis. Hence, investors might require a premium to hold assets that have high tail risk. Indeed, recent asset pricing studies have demonstrated that such a premium exists and is economically significant in explaining cross-sectional stock returns.

Kraus and Litzenberger [1976] show that investors dislike stocks with high tail risk in terms of negative co-skewness, and that stocks with more negative co-skewness tend to have higher expected returns. Harvey and Siddique [2000] show that co-skewness is economically important and commends an average risk premium of 3.6% per year for U.S. stocks.

Bawa and Lindenberg [1977] suggest that a natural extension of the CAPM, one that takes into account the asymmetric risk preference, is to specify asymmetric downside and upside betas. Ang, Chen, and Xing [2006] estimate that the downside risk premium is approximately 6% annually for U.S. stocks.

Most previous studies on the low-volatility anomaly and downside risk premium were conducted on the U.S. stock universe. It appears that volatility is not compensated, and that a downside risk premium is economically significant in cross-sectional U.S. stocks. On the other hand, there is little related research on the equity fund universe. One exception is Moreno and Rodríguez [2009], who found evidence that adding a co-skewness factor is economically and statistically significant in evaluating mutual funds.

The biggest difference between the fund universe and the stock universe is the idiosyncratic component. Funds have eliminated most idiosyncratic risks from individual stocks, because a typical mutual fund holds more than 100 stocks, and diversification significantly lowers their idiosyncratic component. However, funds still have systematic risks related to the nature of their investments, causing them to have different tail risks.

Another difference for the fund universe is that transaction costs are fully considered, as all our analyses are based on net total fund returns. Estimating transaction costs in the stock universe, by contrast, can be very challenging.

We show that the low-volatility anomaly indeed appears in both U.S. equity funds and non-U.S. equity funds. However, the tail-risk premium is economically significant in both U.S. equity funds and non-U.S. equity funds.

DATA DESCRIPTION

We use Morningstar's open-end equity mutual fund universe, containing both alive and dead funds, from January 1980 to September 2011. We collect only the oldest share class for each fund. We include both U.S. and non-U.S. equity mutual funds. Morningstar categories include all funds from the nine size-valuation style boxes that form the U.S. equity universe, although we focus on the somewhat less granular valuation-based columns from the style box (value, core, and growth), size-based rows from the style box (large, mid, and small), plus the non-U.S. category, as we analyze subsections of the U.S. equity universe. This helps ensure that each composite contains a reasonably large number of funds. Most of the non-U.S. equity funds have inception dates later than 1990.

We collected 3,389 U.S. equity funds and 1,055 non-U.S. equity funds, all with at least a five-year history. The appendix shows that the majority of the equity funds exhibit non-normality, which indicates that tail risk is an important consideration, in addition to volatility.

VOLATILITY VS. TAIL RISK

In this section we test three risk metrics: volatility (VOL), skewness (SKEW), and excess conditional-value-at-risk (ECVaR).

Volatility is simply the standard deviation of the fund's total returns. It is not sensitive to the tail information—but both SKEW and ECVaR are tail-risk measures.

SKEW is a measure of the data's asymmetry around the sample mean. It is the third standardized moment. Negative skewness indicates a greater probability for large negative returns.

The second Tail-risk measure, ECVaR, specifically measures the left-tail risk. The right tail affects skewness, but has no impact on ECVaR. ECVaR is based on conditional value at risk (CVaR, also known as expected tail loss). ECVaR is the fund's CVaR, in excess of the implied CVaR, with a normal distribution and the same mean and standard deviation in a given period. In other words, the fund's ECVaR is a version of CVaR, normalized by controlling for fund volatility. See the appendix for more details on ECVaR.

We compute the risk measures (VOL, SKEW, and ECVaR) on total returns for most of the following analyses. Total return can be decomposed into systematic and idiosyncratic components. In the final section, "Beta vs. Co-Skewness," we investigate the systematic components of both volatility (beta) and tail risk (co-skewness) to see whether beta or co-skewness is compensated.

Exhibit 1 shows the average correlation among the three risk measures for both U.S. and non-U.S. equity funds. The sample period is from January 1980 to September 2011. The correlation is measured across all the U.S. and non-U.S. equity funds using a rolling five-year window, averaged over time.

VOL has relatively low correlation with both SKEW and ECVaR in both fund universes, indicating that VOL captures different information from the left tail. ECVaR depends on the tail level; a smaller tail level corresponds to a more extreme left tail. Exhibit 1's ECVaR values are computed with a tail level of 5%, i.e., the worst 5% of returns. The correlation between ECVaR and SKEW is 66% and 74% for U.S. and non-U.S. equity funds, respectively, indicating that ECVaR and SKEW capture much of the same left-tail information. We focus more on ECVaR, because our empirical analyses show that ECVaR performs slightly better than SKEW.

EXHIBIT 1

The Average Correlation for Volatility and Tail-Risk Measures (January 1980–September 2011)

	U.S. Equity Funds			Non-U.S. Equity Funds		
	Volatility	**Skewness**	**ECVaR**	**Volatility**	**Skewness**	**ECVaR**
Volatility	1.00	0.13	–0.10	1.00	0.22	0.01
Skewness		1.00	0.66		1.00	0.74
ECVaR			1.00			1.00

EXHIBIT 2

Summary Statistics for the Five VOL Quintiles and Five ECVaR Quintiles (January 1985—September 2011)

Panel A: U.S. Equity Funds

VOL	**Low Volatility 1**	**2**	**3**	**4**	**High Volatility 5**
Ari. Mean	4.87%	5.70%	6.41%	6.36%	6.51%
Geo. Mean	4.03%	4.57%	5.09%	4.76%	4.03%
Std. Dev.	12.57%	14.54%	15.64%	17.23%	21.46%
Sharpe Ratio	0.39	0.39	0.41	0.37	0.30
ECVaR	**Low Tail Risk 1**	**2**	**3**	**4**	**High Tail Risk 5**
Ari. Mean	4.30%	5.64%	6.24%	6.71%	6.97%
Geo. Mean	2.83%	4.34%	4.94%	5.37%	5.43%
Std. Dev.	16.59%	15.56%	15.54%	15.77%	16.87%
Sharpe Ratio	0.26	0.36	0.40	0.43	0.41

Panel B: Non-U.S. Equity Funds

VOL	**Low Volatility 1**	**2**	**3**	**4**	**High Volatility 5**
Ari. Mean	6.21%	5.39%	5.75%	5.27%	4.86%
Geo. Mean	5.18%	4.05%	4.22%	3.58%	2.46%
Std. Dev.	13.80%	15.77%	16.85%	17.69%	21.22%
Sharpe Ratio	0.45	0.34	0.34	0.30	0.23
ECVaR	**Low Tail Risk 1**	**2**	**3**	**4**	**High Tail Risk 5**
Ari. Mean	3.83%	4.89%	6.01%	6.42%	6.33%
Geo. Mean	2.28%	3.45%	4.59%	4.90%	4.42%
Std. Dev.	17.12%	16.41%	16.20%	16.77%	18.73%
Sharpe Ratio	0.22	0.30	0.37	0.38	0.34

VOLATILITY PREMIUM VERSUS TAIL-RISK PREMIUM

Our portfolio formation methodology follows Harvey and Siddique [2000]. Using the first 60 months of returns, we compute the VOL and ECVaR for each of the equity funds. We sort all the equity funds into quintiles based on VOL or ECVaR, then average the 61-month excess returns with equal weights for each quintile. The first quintile (Q1) is the proxy for the lowest VOL or lowest tail-risk (ECVaR) quintile, and the fifth quintile (Q5) is the proxy for the highest VOL or highest tail-risk (ECVaR) quintile. A more negative ECVaR value means a more severe loss, and therefore higher tail risk.

One problem associated with tail-risk estimates is the trade-off between statistical significance and survivorship bias. We need at least 60 months of data to estimate tail risk, which unfortunately introduces some survivorship bias, because some funds with the worst losses were shut down.

Exhibit 2 presents the summary statistics for the five VOL quintiles and five ECVaR quintiles for U.S. and non-U.S. equity funds. All returns in this article are annualized and in excess of the Treasury bill rate. We compute the volatility premium and tail-risk premium as the difference in arithmetic mean for Q5 and Q1 (Q5 – Q1), as shown in Exhibit 2.

At first glance, the volatility premium is positive for U.S. equity funds (1.64%). But on a risk-adjusted basis, the highest volatility quintile (Q5) has the lowest Sharpe ratio, significantly lower than that of the other four quintiles. The geometric mean for Q1 and Q5 is almost the same from January 1985 to September 2011, with both at 4.03%. This provides evidence that the volatility is not compensated on a risk-adjusted basis. For non-U.S. equity funds, the volatility premium (Q5 – Q1) is negative (–1.35%), which strongly supports the low-volatility anomaly.

However, the tail-risk premium (Q5 – Q1) for both U.S. (2.67%) and non-U.S. equity funds (2.50%) are positive, even after we control for volatility. This suggests that tail risk is compensated.

Exhibit 3 shows the risk-return relationship for the five VOL quintiles and five ECVaR quintiles for U.S. equity funds presented in Exhibit 2, Panel A. The VOL quintiles have a positive slope from Q1 to Q3, but are flat from Q3 to Q5. We can partially explain the positive slope from Q1 to Q3 with the possibility that Q1 equity funds hold relatively higher allocations to cash and bonds than do funds in Q3, so that Q1 has lower volatility and lower arithmetic return. On the other hand, the five ECVaR quintiles present a very different picture. They have a similar level of standard deviations by construction, but the expected excess returns are significantly lower for the lowest tail risk quintile, indicating that the tail-risk premium is significantly positive.

We measure ECVaR through a five-year period, and the left tail has only three monthly data points when the tail level is 5% ($0.05 * 60 = 3$). To address concerns over the small number of data points, we also plotted the tail-risk frontier with a tail level of 10% in Exhibit 3 (triangles). We found that the tail-risk frontier here is similar to the same frontier for the 5% tail level, except that the tail-risk premium for the 10% level is a little lower.[1]

Exhibit 4 shows the risk-return relationship for the five VOL quintiles and five ECVaR quintiles, at both the 5% and 10% tail levels, for non-U.S. equity funds. The five VOL quintiles have a negative slope, which

EXHIBIT 3

The Risk-Return Relationship for the Five VOL Quintiles and Five ECVaR Quintiles (Both 5% and 10% Level)

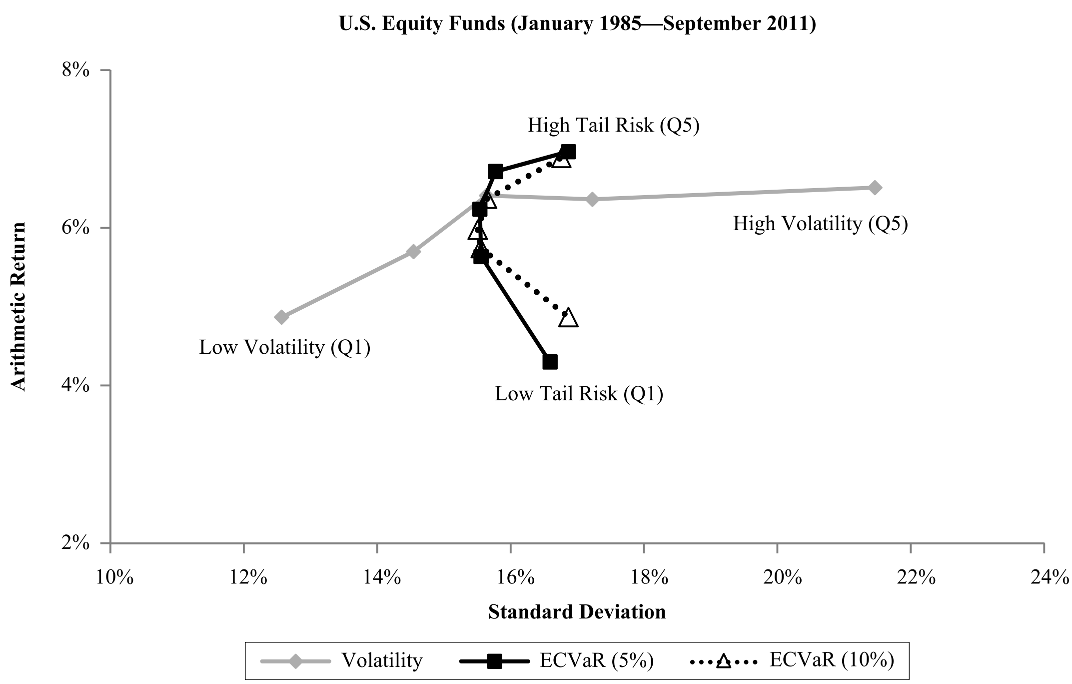
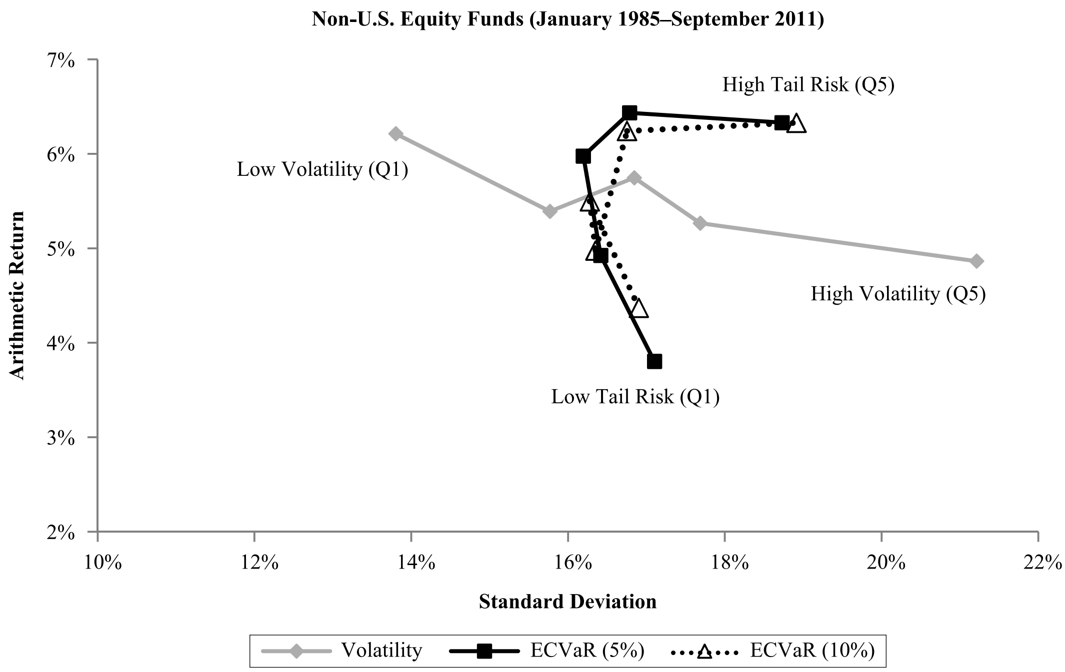


Exhibit 4

The Risk-Return Relationship for the Five Volatility Quintiles and Five ECVaR Risk Quintiles (Both 5% and 10% Levels)



confirms the low-volatility anomaly. On the other hand, the five ECVaR quintiles at both the 5% and 10% tail levels confirm that the tail-risk premium (Q5 – Q1) is positive.

Exhibit 5 shows the Q1 and Q5 value growth for both VOL and ECVaR (5% tail level) for U.S. equity funds. For the entire period, the highest tail-risk composite dominated the lowest tail-risk composite. However, the highest-volatility composite dominated the lowest-volatility composite only for the first half of the period, mostly during the technology bubble. The highest-volatility composite underperformed the lowest volatility composite after the technology bubble burst, so they ended with almost the same value. This result is consistent with the observation of Baker et al. [2011] that the low-volatility anomaly widened after the technology bubble burst.

Exhibit 6 shows the growth of value for Q1 and Q5 for both VOL and ECVaR (5% tail level) for non-U.S. equity funds. For the entire period, the highest tail-risk composite dominated the lowest tail-risk composite. In contrast, the lowest-volatility composite dominated the highest-volatility composite for the entire period.

Exhibit 5

Growth of $1 for Selected Volatility and Tail-Risk Quintiles (ECVaR for 5% Tail Level)

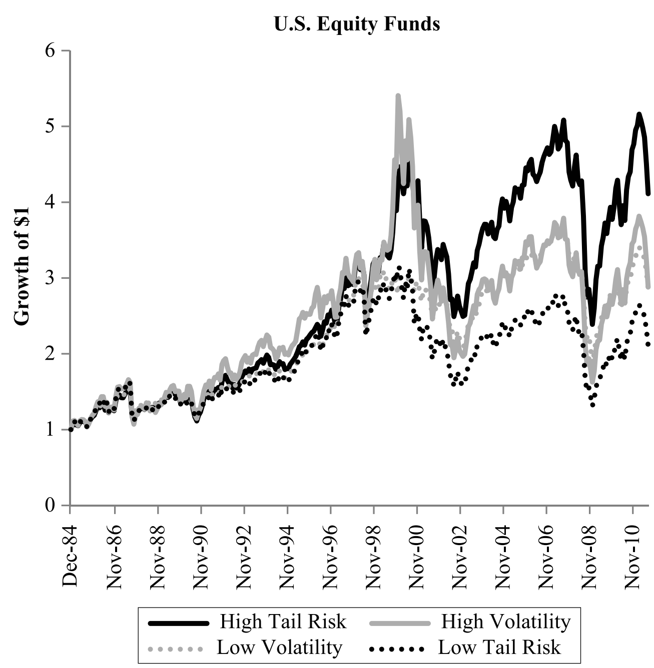


EXHIBIT 6

Growth of $1 for Selected Volatility and Tail-Risk Quintiles

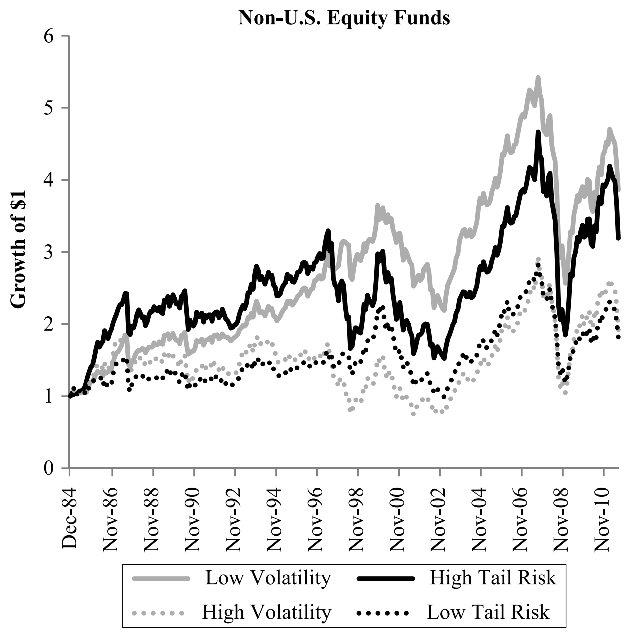


EXHIBIT 7

Volatility Alpha and Tail-Risk Alpha for U.S. and non-U.S. Equity Funds from January 1985 to September 2011

	Carhart Alpha	*t*-stat	Category Average Alpha	*t*-stat
Panel A: Volatility Alpha				
All U.S. Equity	–0.25%	–0.26	–1.59%	–0.88
Large	0.66%	0.83	–0.71%	–0.56
Medium	–0.24%	–0.18	–2.57%	–1.39
Small	–1.71%	–1.27	–4.77%	–2.32
Core	–1.75%	–3.03	–1.68%	–1.62
Growth	–0.13%	–0.13	–1.61%	–1.02
Value	0.24%	0.33	0.43%	0.45
Non-U.S. Equity	–1.61%	–0.92	–3.61%	–2.21
Panel B: Tail-Risk Alpha				
All U.S. Equity	2.65%	2.82	2.44%	2.63
Large	1.87%	2.42	1.31%	1.70
Medium	2.14%	1.95	1.88%	1.59
Small	2.25%	1.89	3.12%	2.35
Core	1.67%	2.45	1.89%	2.51
Growth	1.41%	1.57	1.01%	1.12
Value	1.82%	2.48	2.10%	2.57
Non-U.S. Equity	2.82%	1.70	1.79%	1.12

Exhibits 5 and 6 both support the thesis that volatility is not compensated, but tail risk is compensated, in both U.S. and non-U.S. equity mutual funds.

PORTFOLIO PERFORMANCE OF (Q5 – Q1)

Alpha is an important element in performance evaluation, and so we are interested in measuring the alpha of the long/short (Q5 – Q1) portfolio on a risk-adjusted basis for both VOL (high-volatility minus low-volatility risk) and 5% tail-level ECVaR (high tail risk minus low tail risk). Exhibit 7 shows the regression results. We measured the alpha of Q5 – Q1 against two benchmarks from January 1985 to September 2011: the Carhart [1997] four-factor model (the three Fama-French factors plus momentum),[2] and the fund category average. The *t*-statistics of alpha are simply the *t*-statistics of the regression intercept. The differences between Panel A and Panel B of Exhibit 7 are striking.

For VOL in Panel A, 13 of the 16 alphas are negative, none of the alphas are significantly positive, and the Carhart alpha for the core category is even significantly negative at the 1% level. The results are consistent with our main thesis: that volatility is not compensated.

In contrast, for ECVaR in Panel B, all 16 alphas are positive, and half of the alphas are significant at the 5% level. The Carhart alpha of the Q5 – Q1 portfolio for all U.S. equity funds is 2.65% per year, with a *t*-statistics of 2.82. This confirms that the tail-risk premium is robust, after adjusting for the Carhart [1997] four-factors.

FAMA–MACBETH REGRESSIONS

Now we examine the influence of tail risk on the cross-section of fund returns through standard Fama and MacBeth [1973] regressions. In each month, from January 1985 to September 2011, we run a simple cross-sectional regression of future fund returns on past volatility and tail risk. We use the past 60 months of data to estimate each fund's volatility or tail risk each month. For example, we use data from January 1980 to December 1984 to estimate the volatility or tail risk for each mutual fund for the first holding period, i.e., January 1985. This gives us 321 monthly estimates of the slope coefficients, along with the associated standard errors. We then aggregate these slope coefficient estimates across time.

Exhibit 8 shows the average slopes and associated *t*-statistics (in parentheses). As expected, none of the VOL coefficients are positively significant. The ECVaR

EXHIBIT 8

Fama–MacBeth Regressions for U.S. Equity Funds and Non-U.S. Equity Funds (January 1985–September 2011)

	All U.S.	Large	Medium	Small	Core	Growth	Value	Non-U.S.
VOL	0.04	0.06	0.05	–0.03	–0.02	0.04	0.05	–0.03
	(0.60)	(0.96)	(0.82)	(–0.43)	(–0.36)	(0.80)	(0.93)	(–0.62)
ECVaR	–0.08	–0.06	–0.04	–0.05	–0.09	–0.01	–0.06	–0.06
	(–2.79)	(–2.17)	(–1.46)	(–1.81)	(–3.55)	(–0.48)	(–2.41)	(–1.51)

is statistically significant at the 5% level for the ALL U.S. equity funds category, and for three other categories.[3] Note that the negative coefficient for ECVaR is interpreted as a more negative or severe ECVaR (i.e., higher tail risk), which corresponds with a higher expected return.

In summary, our Fama–MacBeth analyses are consistent with previous results. We find that overall, the tail risk is significantly related to future fund returns. It is largely significant, when we control for size, value, fund beta, and fund momentum.

FUND BETA VS. CO-SKEWNESS

So far we have focused our study on the VOL that is measured on total returns, not on a systematic component or market risk, such as fund beta. ECVaR and SKEW are also measured on total returns. In this section, we test whether the systematic component of both volatility and tail risk tell the same story as total returns. The systematic variables for VOL and SKEW are fund beta and co-skewness, respectively. Co-skewness is the component of an asset's skewness related to the market portfolio's skewness. Although fund beta seems like a clear corollary for a systematic risk measure associated with VOL, an equivalent systematic risk measure for ECVaR is somewhat less clear. Of the potential measures, co-skewness seems the most logical. Harvey and Siddique [2000] define the standardized unconditional co-skewness as

$$Co\text{-}skewness = \frac{E[\varepsilon_{i,t}\varepsilon_{M,t}^2]}{\sqrt{E[\varepsilon_{i,t}^2]}\,E[\varepsilon_{M,t}^2]}$$

$$\varepsilon_{i,t} = r_{i,t} - r_{ft} - \beta_i(r_{M,t} - r_{ft})$$

$$\varepsilon_{m,t} = r_{M,t} - avg(r_{M,t})$$

where the residual $\varepsilon_{i,t}$ is computed from the regression of the excess return on the contemporaneous market's excess return for security i in period t. $\varepsilon_{m,t}$ is the market return in period t in excess of average market return. A negative co-skewness means that the security is adding negative skewness to the portfolio. Harvey and Siddique [2000] suggest that a stock with more negative co-skewness should have a higher expected return.

Exhibit 9 plots the risk-reward tradeoff for fund beta and co-skewness for U.S. equity funds. On a risk-adjusted basis, fund beta is not compensated, because the Sharpe ratio for Q5 is the lowest. On the other hand, the co-skewness premium (Q5-Q1) is 1.85%, a little lower than the ECVaR premium (2.67%). Exhibit 10 shows the growth of value for Q1 and Q5, for both VOL and Co-skewness, for U.S. equity funds. The findings in Exhibits 9 and 10 are largely consistent with Exhibit 3 and Exhibit 5, respectively. Beta and co-skewness analyses for non-U.S. equity funds yield similar results.

CONCLUSIONS

We investigated whether volatility and tail risk are compensated in both U.S. and non-U.S. equity mutual funds. We have provided evidence that volatility is not compensated on a risk-adjusted basis for both U.S. equity funds and non-U.S. equity funds. Our research using the fund universe largely supports similar findings in cross-sectional stocks. On the other hand, the tail-risk premium in both U.S. and non-U.S. equity mutual funds is economically significant. Funds that have higher tail risk have higher expected returns. This tail risk–return relationship is consistent with an economy in which agents demand a premium to compensate for tail risk.

We introduced a new left tail-risk measure, excess conditional-value-at-risk (ECVaR), and also used a standard measure: skewness (SKEW). The cross-sectional premium for bearing tail risk is approximately 2.67% per annum for U.S. equity funds, and 2.50% for non-U.S. equity funds, with the ECVaR measure. The

EXHIBIT 9

The Risk–Return Relationship for the Five Beta Quintiles and Five Co-Skewness Quintiles

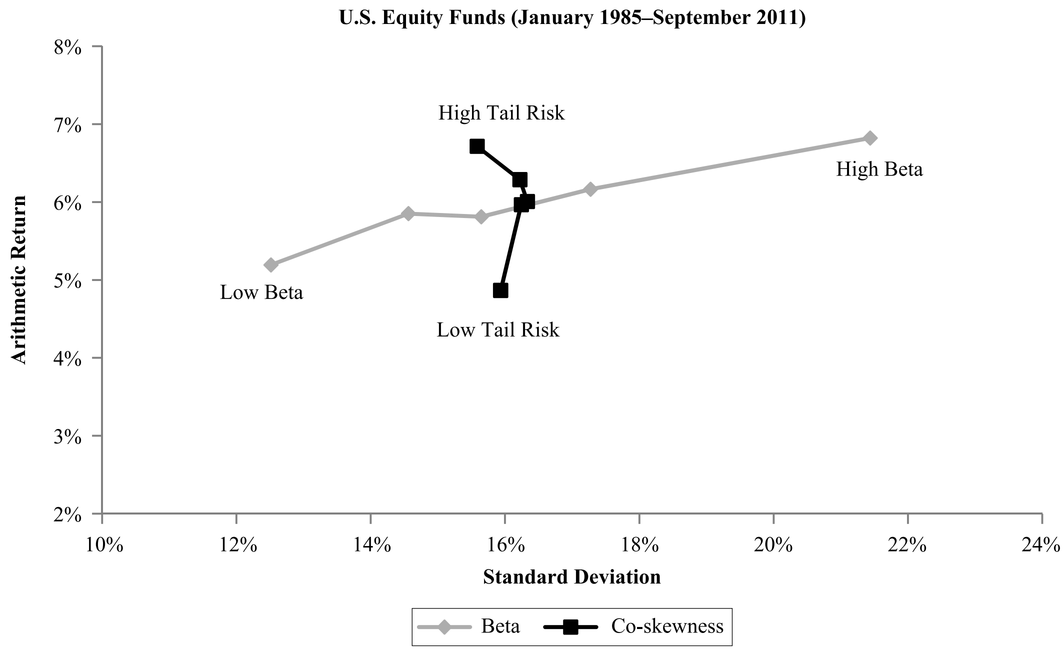
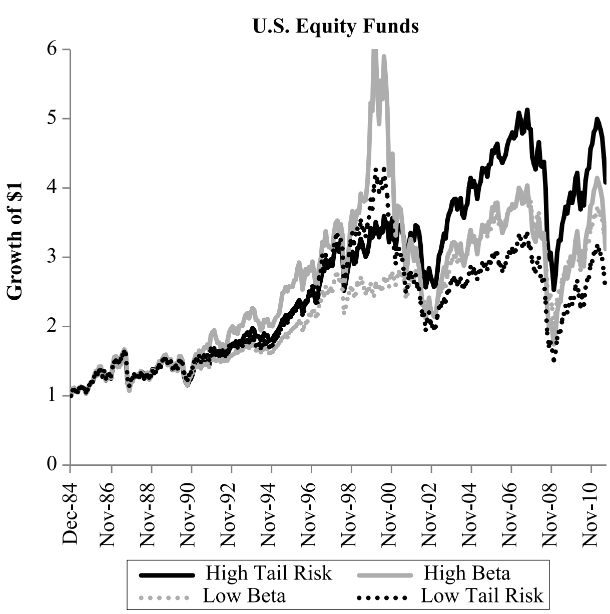


EXHIBIT 10

Growth of $1 for Selected Beta and Tail-Risk (Co-Skewness) Quintiles



tail-risk premium remains statistically significant after controlling volatility as well as size, value, fund beta, and fund momentum for equity funds.

Finally, analyses on fund beta and co-skewness, the systematic variables for volatility and tail risk, respectively, confirmed the main thesis that volatility is not compensated, but tail risk is compensated.

APPENDIX

NON-NORMALITY OF EQUITY MUTUAL FUNDS

It is well known that the returns of many financial asset classes do not follow a normal distribution. Mandelbrot [1963] found that changes in cotton prices did not follow a normal distribution, but instead a stable distribution. Fama [1965] confirmed the non-normality in stock prices. Since then, there has been extensive literature on non-normality of financial assets. For an introduction, see Rachev et al. [2005].

Normal distribution can underestimate tail risk considerably. We can easily test whether or not equity mutual funds behave the same. Exhibit A1 shows that U.S. and non-U.S. equity funds clearly exhibit non-normality. The median

EXHIBIT A1

Non-Normal Tests for U.S. and Non-U.S. Equity Mutual Funds

	Number of Funds	Median Skewness	Median kurtosis	Non-Normal Test
All U.S. Equity	3389	–0.59	4.35	84.21%
Large	1992	–0.62	4.28	84.54%
Medium	668	–0.59	4.69	86.98%
Small	729	–0.52	4.24	80.80%
Core	1275	–0.64	4.32	85.88%
Growth	1392	–0.49	4.19	79.45%
Value	722	–0.67	4.67	90.44%
Non-U.S. Equity	1055	–0.62	4.54	84.17%

skewness and kurtosis for U.S. equity funds are −0.59 and 4.35, respectively, well beyond that of a normal distribution. The Jarque–Bera [1980] tests show that the normal distribution is rejected for about 84% of both U.S. and non-U.S. equity funds that have at least a five-year history. For each particular style or size, about 80% to 90% of funds exhibit non-normality.

ECVaR

We are interested in a measure that only measures the left tail risk. Our new measure is based on conditional value at risk (CVaR). The precursor to CVaR, and a slightly better known measure of downside risk, is the standard value-at-risk (VaR) measure. VaR estimates the loss that we expect will be exceeded, with a given level of probability over a specified time period. CVaR is closely related to VaR and is calculated by taking a probability-weighted average of the possible losses, conditional on the loss being equal to or exceeding the specified normal-distribution VaR. Other terms for CVaR include mean shortfall, tail VaR, and expected tail loss. CVaR is a comprehensive measure of the entire part of the tail being observed, and for many is the preferred measurement of tail risk.

For most of our study, we fixed the CVaR probability level at 5% (corresponding to a confidence level of 95%). For a normal distribution with mean of μ and standard deviation of σ, the CVaR has a closed-form formula (see Rockafellar and Uryasev [2000]):

$$CVaR_{normal} = (\mu - 2.06 * \sigma)$$

ECVaR for a non-normally distributed fund is defined as:

$$ECVaR = CVaR_{fund} - (\mu - 2.06 * \sigma)$$

It is the difference between the fund's $CVaR_{fund}$ and the $CVaR_{normal}$, under the assumption that the fund's return distribution is normally distributed. ECVaR controls for the fund's volatility, or beta, by subtracting $CVaR_{normal}$. The constant coefficient—2.06 in the previous equations—is replaced by 1.76 for a tail level of 10%.

ENDNOTES

[1]Unfortunately, a higher tail level in the ECVaR measure captures a higher portion of the bulk distribution, and so captures the volatility effect. Daily returns mitigate the small sample issue, but are not available for a longer period of time.

[2]For U.S. equity funds, we use and download the three Fama–French factors and momentum factor from the Kenneth French Data Library. For non-U.S. equity funds, we use the three global Fama–French factors and global momentum factor. The market factor is adjusted for average fund expense.

[3]In unreported results, the *t*-statistics are largely unchanged for ECVaR, when we add beta and momentum as independent variables into the regression.

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