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Constructing Equity Market–Neutral VIX Portfolios with Dynamic CAPM

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Market-neutral hedge funds typically try to exploit pricing differences between two or more related securities to generate performance that is uncorrelated with a target benchmark. This may involve creating corresponding long and short positions in related securities. Such strategies may fail because the related securities used in the strategy do not behave as expected, causing the fund's portfolio to become excessively correlated with the benchmark. For instance, using broad measures of the equity market as target benchmarks, Patton [2009] found that many hedge funds that are so-called *equity market neutral* (EMN) fail to achieve statistically significant neutrality.

We propose an algorithmic remedy to this problem. We apply Kalman filter–based dynamic regression to generate daily updates of portfolio weightings to create an EMN portfolio. To demonstrate the method, we apply it to Chicago Board Options Exchange (CBOE) Volatility Index (VIX) futures indexes. These indexes are specified and structured in terms of traded VIX futures contracts. Their specifications and structures are public information, which makes them investable. We apply our portfolio construction method to the two most popular indexes, the VIX short-term and midterm futures indexes. We then apply this method to combinations of the short-term futures index and other VIX futures indexes to demonstrate the usefulness and generality of the approach.

The next section discusses the VIX futures indexes and key statistics. The following section discusses the dynamic capital asset pricing model (CAPM) and how it could be employed to estimate the derivatives' Greeks. The next two sections present an operational EMN portfolio construction method and robustness checks. The last section offers our conclusions.

VIX FUTURES INDEXES AND KEY STATISTICS

Whaley [1993] developed the VIX Index, and its construction was refined by the CBOE in 2003. The index is a weighted blend of prices for a range of S&P 500 Index options that are traded on the CBOE. The VIX Index itself is not tradable, but the CBOE began trading futures contracts on the VIX on March 26, 2004. Exhibit 1 shows open interest for VIX futures contracts for all expirations from that date to June 2015. The exhibit reveals a substantial increase in open interest over the period.

Since the introduction of VIX futures, various VIX–related instruments have been formulated (see Alexander and Korovilas [2013]). Researchers and practitioners have become increasingly interested in VIX derivatives in terms of their use in both

Exhibit 1
CBOE VIX Futures Open Interest (March 26, 2004–June 30, 2015)

Source: CBOE.

(1) volatility selling (see Carr and Wu [2009]; Rhoads [2011]; Sinclair [2013]) and (2) portfolio protection strategies (Dash and Moran [2005]; Chen, Chung, and Ho [2011]; Mencía and Sentana [2013]).

S&P developed the VIX futures indexes and has made them available as daily data since December 20, 2005. The indexes are computed as returns of combinations of VIX futures contracts, which are rolled daily between contract expiration days. The VIX Short-Term Futures Index (Bloomberg: SPVXSP) rolls from the one-month VIX futures contract into the two-month contract, maintaining a constant one-month maturity; the VIX two-month futures index (Bloomberg: SPVIX2ME) rolls from the two-month futures into the three-month futures, maintaining a constant two-month maturity; and so on, out to the three-month index (Bloomberg: SPVIX3ME) and the four-month index (Bloomberg: SPVIX4ME). The underlying holdings for the VIX Mid-Term Futures Index (Bloomberg: SPVXMP) are the four- through seven-month VIX futures. The index continuously rolls out of the four-month VIX futures and into the seven-month futures, maintaining a constant five-month maturity. The VIX 6-Month Futures Index (Bloomberg: SPVIX6ME) uses the five- through eight-month VIX futures, rolling out of the five-month

futures and into the eight-month futures, maintaining a constant six-month maturity.

VIX futures indexes are, of course, highly correlated with the VIX Index. In addition, VIX futures indexes are affected by roll costs or roll gains in rolling from shorter contracts to longer contracts, depending on whether the term structure in the VIX futures market is in contango or backwardation, respectively. Exhibit 2 presents summary statistics for the returns of the VIX futures indexes from December 20, 2005, to June 30, 2015.

Exhibit 3 presents the correlation matrix for the returns of the S&P 500 Index and the VIX futures indexes from December 20, 2005, to June 30, 2015.

The six VIX futures indexes are highly negatively correlated with the S&P 500 Index. They are also highly positively correlated with each other because they are driven by the same underlying VIX fluctuations and similar roll costs/gains from the term structure. In addition, the correlation between pairs of futures indexes with the closest maturities is high because their underlying futures are synchronized in their movements.

DYNAMIC CAPM

The portfolio construction method presented in the following equation is rooted in an accurate estimation of

E X H I B I T 2
Return Statistics for the S&P 500 Index and VIX Futures Indexes (December 20, 2005–June 30, 2015)

	S&P 500	SPVXSP	SPVIX2ME	SPVIX3ME	SPVIX4ME	SPVXMP	SPVIX6ME
Annual mean (%)	5.33	–43.00	–28.73	–18.59	–16.66	–13.92	–10.84
Volatility (%)	20.89	61.97	46.96	38.78	34.02	31.12	28.95
Sharpe ratio	0.19	–0.93	–0.75	–0.56	–0.57	–0.52	–0.44
Skewness	–0.33	0.57	0.40	0.44	0.44	0.46	0.49
Excess kurtosis	10.52	3.01	2.65	2.83	3.07	3.28	3.49
Total return (%)	63.79	–99.52	–96.00	–85.85	–82.32	–75.96	–66.41
Max drawdown (%)	56.78	99.81	98.59	95.35	93.44	90.64	86.46

E X H I B I T 3
Correlation Matrix for Returns on S&P 500 Index and VIX Futures Indexes (December 20, 2005–June 30, 2015)

	S&P 500	SPVXSP	SPVIX2ME	SPVIX3ME	SPVIX4ME	SPVXMP	SPVIX6ME
S&P 500	1.00	–0.76	–0.75	–0.76	–0.76	–0.75	–0.73
SPVXSP	–0.76	1.00	0.96	0.95	0.92	0.91	0.88
SPVIX2ME	–0.75	0.96	1.00	0.97	0.95	0.93	0.91
SPVIX3ME	–0.76	0.95	0.97	1.00	0.98	0.97	0.95
SPVIX4ME	–0.76	0.92	0.95	0.98	1.00	0.99	0.97
SPVXMP	–0.75	0.91	0.93	0.97	0.99	1.00	0.99
SPVIX6ME	–0.73	0.88	0.91	0.95	0.97	0.99	1.00

E X H I B I T 4
Static OLS Results for the VIX Futures Indexes (December 20, 2005–June 30, 2015)

	SPVXSP	SPVIX2ME	SPVIX3ME	SPVIX4ME	SPVXMP	SPVIX6ME
α (daily %)	–0.09	–0.05	–0.02	–0.02	–0.02	–0.01
P-value for α	0.07	0.19	0.51	0.42	0.52	0.74
β	–2.27	–1.70	–1.43	–1.25	–1.12	–1.02
P-value for β	0.00	0.00	0.00	0.00	0.00	0.00
R^2 (%)	57.26	56.63	58.18	58.14	56.27	53.72

the parameters of the CAPM. CAPM can be expressed as a static ordinary least squares (OLS) regression as follows:

$$r_i - r_f = \alpha_i + \beta_i * (r_M - r_f) + \varepsilon_i \qquad (1)$$

where r_i is the return for security i; r_f is the risk–free rate; r_M is the market return; ε_i is the residual; and α_i and β_i are parameters. Throughout the article, we use the S&P 500 Index to represent the market portfolio and 90–day Treasury bills to represent the risk–free rate. Exhibit 4 shows the corresponding α and β for each VIX futures index calculated from Equation (1) using static OLS.

However, α and β are hardly constant for the VIX futures indexes. Rolling regression provides one way to capture the dynamics. Exhibit 5 shows the rolling regression-based α and β for the VIX Short-Term Futures Index where the rolling window is 63 market days, corresponding to about three calendar months, and alternatively 126 days, corresponding to about six months. The starting date in the exhibit, March 23, 2006, allows for an initial rolling window of 126 days.

There are major drawbacks associated with using rolling regression to compute α and β. First, as we see in the exhibit, the results are sensitive to the choice of window size, and there is no consensus on choosing

VIX Short-Term Futures Index (March 23, 2006–June 30, 2015)

Panel A: Rolling Regression-Based α

Panel B: Rolling Regression-Based β

63-day ·········· 126-day

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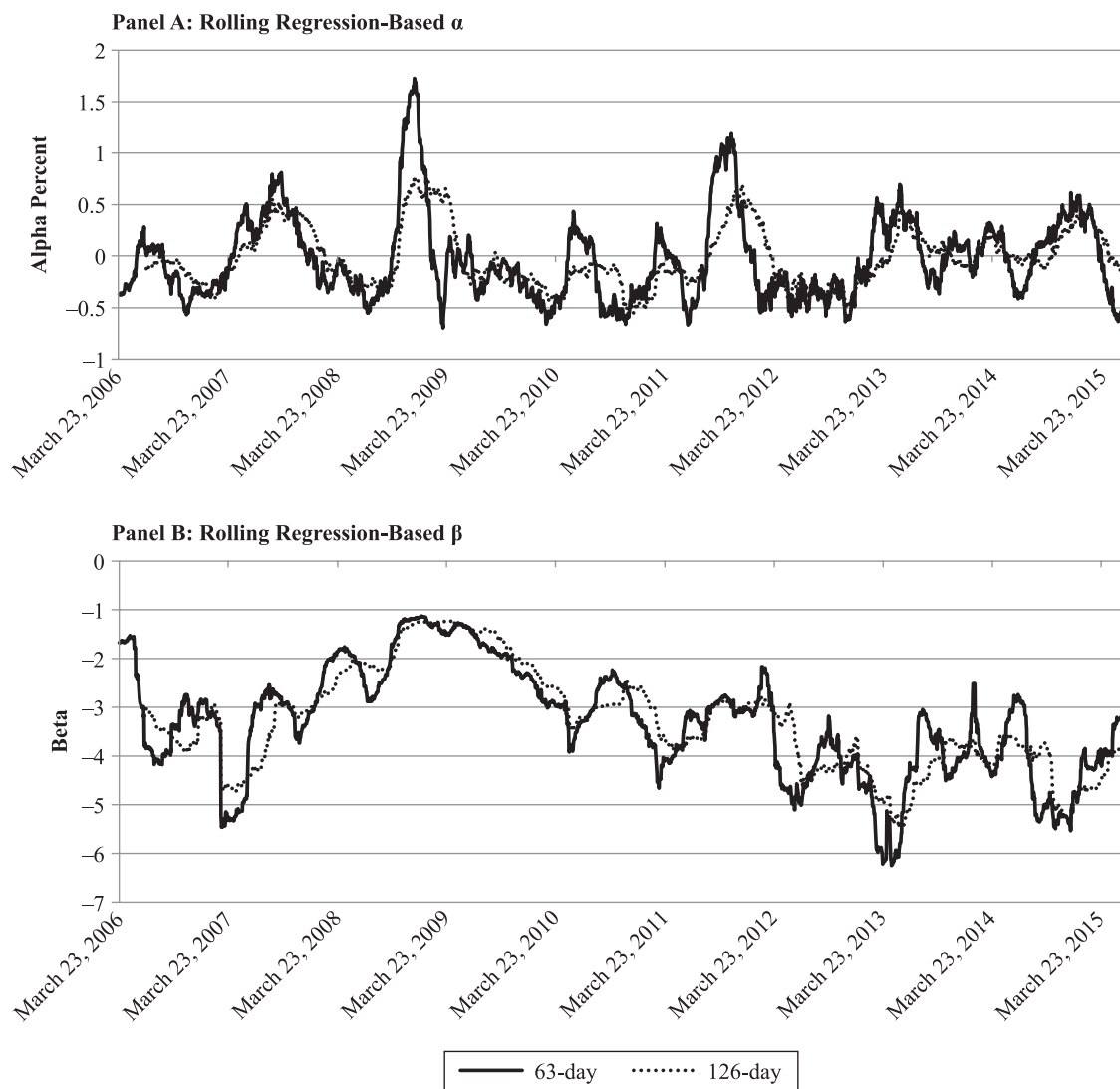
the proper window size—a matter that we find in the following sections to have a large influence on the performance of EMN portfolios. Second, rolling regression is not able to provide instantaneous updates of α and β. Using rolling regression, the so-called *dynamic* α and β are computed over the trailing window. Third, if there are large outliers in the regression, including or removing them could yield very different regression coefficients, which could cause some stability issues when using rolling windows, especially short windows. For example, where the rolling sample moves by one day from an interval of time that includes the outlier

to an interval that excludes the outlier, the estimated coefficients could change abruptly; in reality, they should not. This could induce greater transaction costs and generate incorrect estimates of the weights. Both effects could reduce performance. Outliers could be a frequent occurrence given the volatility of VIX future indexes. Fourth, it is possible that, if we tried all choices of rolling window lengths, we could pick up the optimal window length, but in doing so we could introduce *data snooping* effects.

A dynamic linear model, or *Kalman filter*, provides an alternative way to estimate dynamic α and β.

Here, the state equation is an order-one vector autoregression:

$$\mathbf{x}_t = \phi\mathbf{x}_{t-1} + \mathbf{w}_t. \tag{2}$$

The $p * 1$ state vector $\mathbf{x}_t$ is generated from the previous state vector $\mathbf{x}_{t-1}$ for $t = 1, 2, 3, \ldots, n$. The vector $\mathbf{w}_t$ is a $p * 1$ independent and identically distributed zero-mean normal vector having a covariance matrix of Q. The starting state vector $\mathbf{x}_0$ is assumed to have mean μ_0 and a covariance matrix of $\prod_0$. The state process is a Markov chain, but we do not have direct observation of it. We can only observe a q-dimensional linear transformation of $\mathbf{x}_t$ with added noise. Thus, we have the following observation equation:

$$\mathbf{y}_t = A_t\mathbf{x}_t + \mathbf{v}_t, \tag{3}$$

where A_t is a $q * p$ observation matrix; the observed data are in the $q * 1$ vector $\mathbf{y}_t$; and $\mathbf{v}_t$ is white Gaussian noise with a $q * q$ covariance matrix R. We also assume $\mathbf{v}_t$ and $\mathbf{w}_t$ are uncorrelated. Our primary interest is producing the estimator for the underlying unobserved $\mathbf{x}_t$ given the data $\mathbf{Y}_s = \{\mathbf{y}_1, \ldots, \mathbf{y}_s\}$. In the literature, where $s < t$, this is referred to as *forecasting*; where $s = t$, this is *Kalman filtering*; and where $s > t$, this is *Kalman smoothing*. With the definitions $\mathbf{x}_t^s = E(\mathbf{x}_t|\mathbf{Y}_s)$ and $\mathbf{P}_t^s = E\left[(\mathbf{x}_t - \mathbf{x}_t^s)(\mathbf{x}_t - \mathbf{x}_t^s)'\right]$ and the initial conditions $\mathbf{x}_0^0 = \mathbf{x}_0 = \mu_0$ and $\mathbf{P}_0^0 = \Pi_0$, we have the recursive Kalman filter equations as follows:

$$\mathbf{x}_t^{t-1} = \phi\mathbf{x}_{t-1}^{t-1} \tag{4}$$

$$\mathbf{P}_t^{t-1} = \phi\mathbf{P}_{t-1}^{t-1}\phi' + Q \tag{5}$$

for $t = 1, 2, 3, \ldots, n$ with:

$$\mathbf{x}_t^t = \mathbf{x}_t^{t-1} + K_t(\mathbf{y}_t - A_t\mathbf{x}_t^{t-1}) \tag{6}$$

$$\mathbf{P}_t^t = (I - K_tA_t)\mathbf{P}_t^{t-1} \tag{7}$$

$$K_t = \mathbf{P}_t^{t-1}A_t'(A_t\mathbf{P}_t^{t-1}A_t' + \mathbf{R})^{-1}. \tag{8}$$

Here, $\mathbf{y}_t$ is the vector of observed excess returns of a particular VIX futures index at time t, and A_t is the $1 * 2$ row vector representing the intercept and the excess returns of the S&P 500 Index. The unobserved state $\mathbf{x}_t$ is the vector of time-varying estimates of the parameters of the CAPM equation. As suggested by Lai and Xing [2008], in the case of the dynamic CAPM,

a popular choice is to make ϕ the identity matrix and Q diagonal. Then the Q and R matrixes are estimated using maximum likelihood.

Exhibit 6 presents the CAPM results for the VIX Short-Term Futures Index using the Kalman smoother. As we see in the exhibit, α and β for the VIX futures index are far from constant during the sample period, which suggests that it would be imprudent to build a portfolio based on static CAPM.

We can use the Kalman filter approach to estimate the equivalent option Greeks for VIX derivatives. The profit and loss of an option position over a time period Δt can be decomposed with the Greeks as follows:

$$P\&L_{\Delta t} = \delta * \Delta S + 0.5\Gamma * (\Delta S)^2 + \Theta * \Delta t + \cdots \tag{9}$$

The Greeks for a vanilla option are easily calculated with the Black–Scholes–Merton formula. The VIX futures is a derivative contract of the spot VIX Index, which can also be viewed as a derivative contract on the underlying S&P 500 Index. Thus, we can treat the VIX futures index as a complicated derivative contract on the S&P 500 Index. Since the VIX is a combination of S&P 500 vanilla options, it is possible to develop models for the VIX term structure, and we can estimate the corresponding Greeks for the VIX futures.

Another practical route is to use a fixed Δt (e.g., one day) and estimate the time-varying coefficients in Equation (1). Under such an approximation, δ is directly related to β as $\delta = \beta * VF/S$, where VF is the VIX futures price, S is the S&P 500 Index price, and α combines both the time decay and gamma effects in the derivative contract.

Equation (1) can be directly expanded to include more specific risk factors under different applications. We use the first-order expansion for our EMN portfolio construction because we are more concerned about it being delta/beta neutral. In addition to the Kalman filter–based dynamic CAPM, there are other ways to construct time-varying coefficient models (see Hoover et al. [1998]; Giraitis, Kapetanios, and Yates [2014]) that could serve the same purpose. They are not a panacea for derivative Greeks estimation, but they can at least complement sophisticated pricing models to provide model-free risk estimations.

In the next section, the Kalman filter is used to build an operational dynamic CAPM EMN portfolio.

E X H I B I T 6
VIX Short-Term Futures Index Dynamic Alpha and Beta (December 21, 2005–June 30, 2015)

Panel A: Dynamic Alpha

Panel B: Dynamic Beta
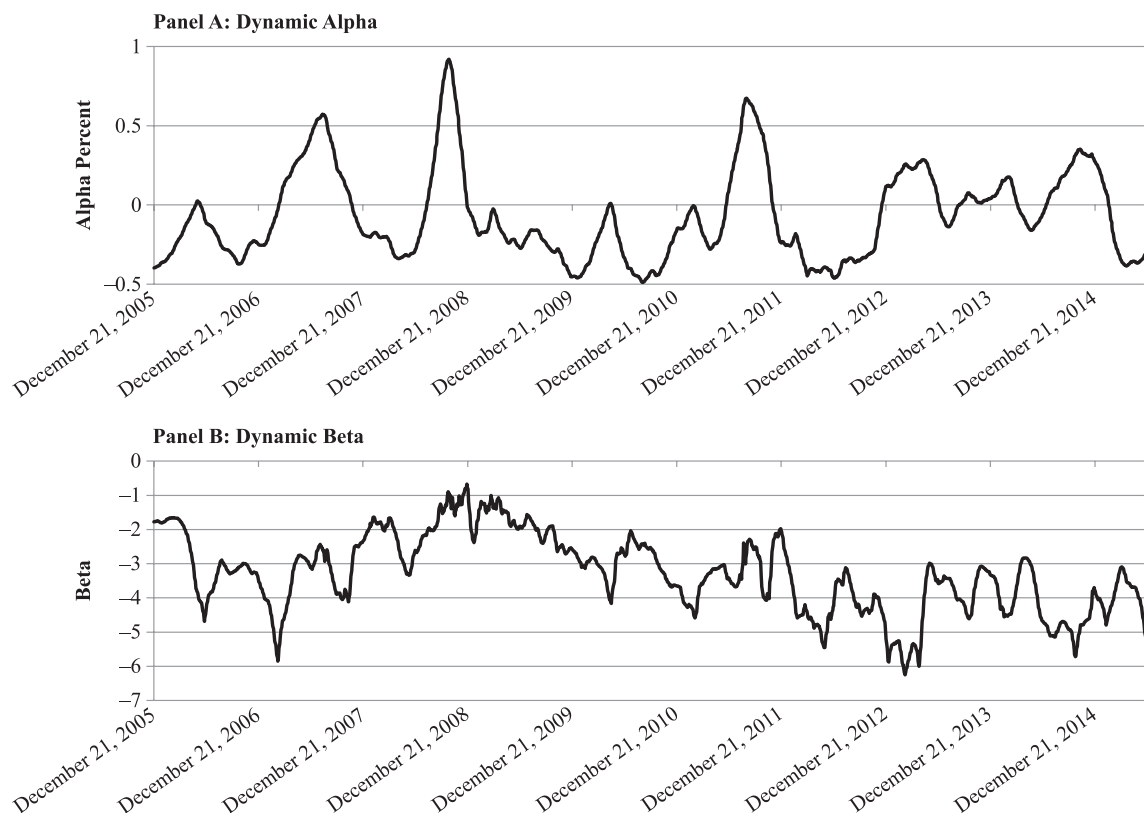
AN OPERATIONAL EMN PORTFOLIO-CONSTRUCTION METHOD

The next portfolio combines positions in the most popular indexes, the VIX short-term and midterm futures indexes. Exchange-traded products linked to these indexes total more than $2 billion in asset value, and trading volume is on a commensurate scale. These securities are highly volatile and thus carry enormous risk. The systematic risk for these securities is high due to the high beta. Meanwhile, the high correlation and similar construction mechanism open ways to hedge market risk, namely the β. By hedging out the β, it is possible to build a pure α generating portfolio (see Figlewski [1984]).

We start with the static CAPM. By assigning proper weights to the securities in the portfolio, the total beta of the portfolio could in principle be reduced to approximately zero. Let $r_{1,t}$, $\alpha_{1,t}$, $\beta_{1,t}$, and $w_{1,t}$ denote, respectively, the daily return, alpha, beta, and portfolio weight of the VIX Short-Term Futures Index on day t. Let $r_{2,t}$, $\alpha_{2,t}$, $\beta_{2,t}$, and $w_{2,t}$ denote the corresponding

measures of the VIX Mid-Term Futures Index on day t. Without using leverage, the summation of the absolute values of the portfolio weights equals 1:

$$\left|w_{1,t}\right| + \left|w_{2,t}\right| = 1. \tag{10}$$

To construct an EMN portfolio, we adjust the weight at the end of each day t so that

$$w_{1,t} * \beta_{1,t} + w_{2,t} * \beta_{2,t} = 0. \tag{11}$$

The weights $w_{1,t}$ and $w_{2,t}$ are opposite in sign, and there are two possible solutions. We use the one that generates a non–negative portfolio alpha $\alpha_{p,t}$ given by

$$\alpha_{p,t} = w_{1,t} * \alpha_{1,t} + w_{2,t} * \alpha_{2,t}. \tag{12}$$

We rebalance the portfolio each day, and the return of the corresponding portfolio on day $t + 1$ becomes

$$r_{p,t+1} = w_{1,t} * r_{1,t+1} + w_{2,t} * r_{2,t+1}. \tag{13}$$

Using the static OLS CAPM for the whole sample, $\alpha = -0.0009$ and $\beta = -2.27$ for the VIX Short-Term Futures Index and $\alpha = -0.0002$ and $\beta = -1.25$ for the VIX Mid-Term Futures Index. Applying Equations (10) and (11) and our rules of computation, we obtain a weight of -0.3551 for the VIX Short-Term Futures Index and a weight of 0.6449 for the VIX Mid-Term Futures Index. Then, from Equation (12), we have an EMN portfolio with $\alpha = 0.0002$.

This ratio of the weights is close to the $-1{:}2$ ratio employed in the UBS XVIX exchange-traded product. Alexander and Korovilas [2013] speculated that this ratio is employed because the VIX Short-Term Futures Index has a long-run volatility approximately twice that of the Mid-Term Index.

Examined from another viewpoint, this concept bears relation to the popular *betting against beta* strategy proposed by Frazzini and Pedersen [2014] (FP hereafter). They observed that, adjusted for the effects of leverage, low-beta stocks outperform high-beta stocks. They argued that many investors are constrained in the degree of leverage they can employ and thus overweight risky high-beta securities. This tendency of investors to buy high-beta securities suggests that low-beta securities tend to be undervalued and thus have higher risk-adjusted returns. FP proposed buying a low-beta portfolio and shorting a high-beta portfolio. The ratio for the long and short weights is simply the inverse ratio of the betas as computed in Equation (11).

The betting against beta strategy may offer insights into how positive alpha is generated in our static CAPM VIX example. That is, near-term futures offer greater portfolio protection during downturns. Investors with limited capital are more inclined to use near-term futures as a directional bet or for portfolio insurance, and the crowded trading in near-term futures would lead to betting against beta in the VIX futures market.

Exhibit 7 shows the behavior of the static CAPM VIX portfolio employing the $-1{:}2$ ratio without using leverage. We set the portfolio value equal to 1 on December 20, 2005, and then chain returns going forward. We call this the *static EMN portfolio*. This portfolio had very good performance until the end of 2010. Afterward, it performed poorly. The problem is the fixed weights. The proposed $-1{:}2$ ratio could work well at times, but there is no guarantee that market structure will remain stable.

Next, we apply rolling regression to calculate α and β, then build an EMN portfolio. On day t, we estimate the CAPM using the past n days of data. Again, we examine two cases, $n = 63$ days and $n = 126$ days. The weights for the EMN portfolio are updated at the end of each rolling window. Exhibit 8 shows the historical performance of the EMN portfolios computed with the 63- and 126-day windows. The starting date in the exhibit is delayed to March 22, 2006, to allow for the 126-day window.

As seen in the exhibit, the two window sizes yield different results. The performance of the portfolio computed from the 126-day window is better from the second half of 2012 onward, which highlights the problem of choice of window size using rolling regression. The rolling window CAPM provides some self-adjusting capability to portfolio construction. This is evident in the fact that performance did not eventually collapse as it did using static regression. We suspect that changes in market structure cause the large drawdowns seen in the exhibit. The rolling approach could adjust to suit the changing environment and generate positive returns again, but the problem lies in part in the choice of window size. If the window size is too short, estimation may be inefficient, degrading portfolio performance. Conversely, if the window size is too large, the effects of changes in market structure may be smoothed over time, subjecting dynamic hedging to the effects of substantial lags that degrade performance. Complicating matters further, market structure may change at varying frequencies. These considerations are arguments for a more dynamic approach.

To this end, we employ the Kalman filter to generate dynamic CAPM results. Cooper [2013] employed the technique using excess returns of the noninvestable VIX Index as the independent variable in the regression. We use S&P 500 Index excess returns as the independent variable. Each day, we use data from the beginning of the sample to estimate dynamic α and β. The weights are calculated according to Equations (10) and (11) and our rules of computation. In effect, we are simulating an out-of-sample portfolio every day. Exhibit 9 shows the performance of the resulting Kalman filter EMN portfolio. Here, the data from December 20, 2005, to February 28, 2006, are used to stabilize the Kalman filter estimation, so the starting date in the exhibit is March 1, 2006.

As seen in the exhibit, there is a strong uptrend in cumulative return. This is an indication of the market neutrality of the portfolio, and it results from the way the Kalman filter updates portfolio weights. The Kalman filter EMN portfolio is based on more timely estimation

Performance of the Static EMN Portfolio Using the VIX Short-Term Futures Index and VIX Mid-Term Futures Index (December 20, 2005–June 30, 2015)

Panel A: Static EMN portfolio

Panel B: Drawdowns
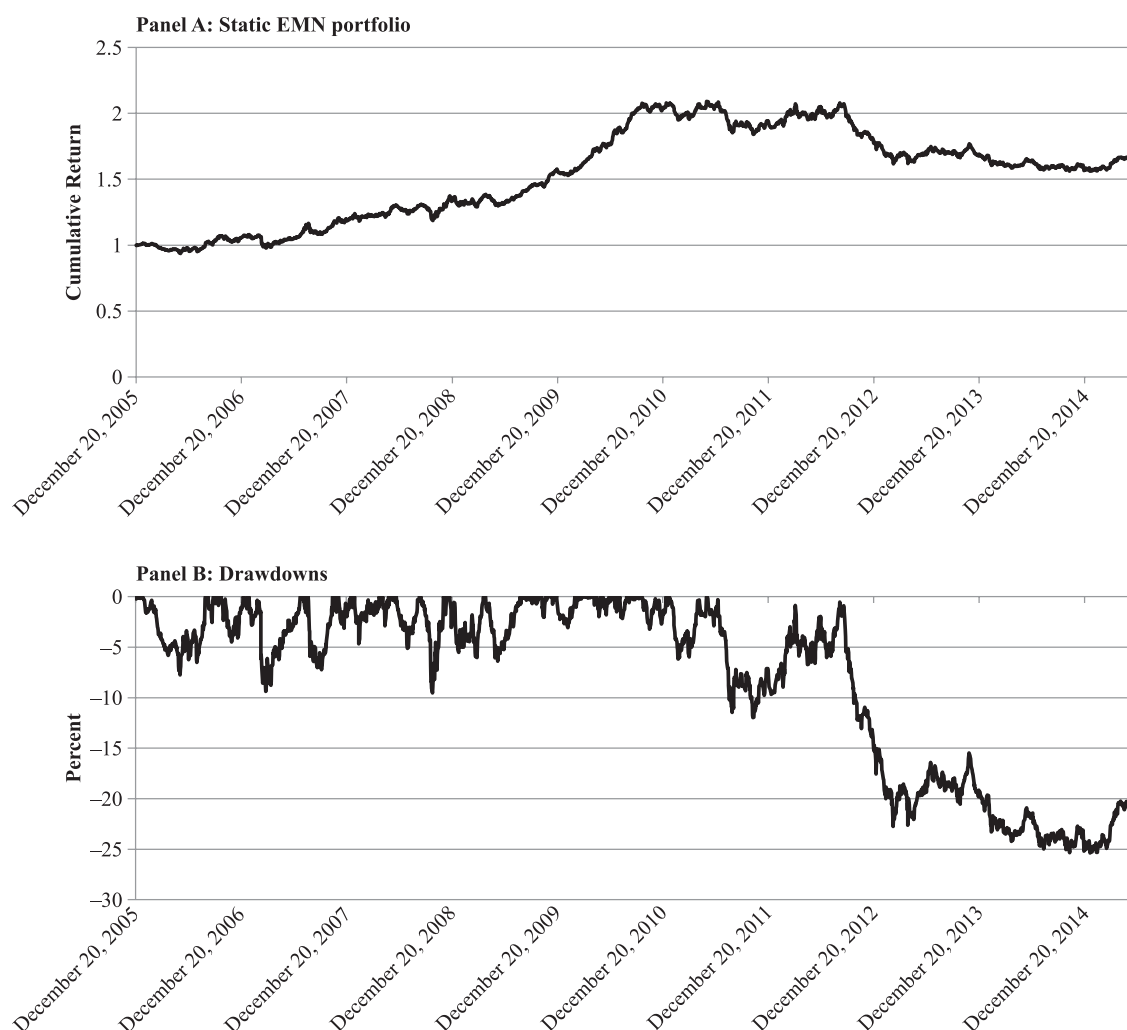
of α and β, which means that the updating of portfolio weights is dynamic and rapid. The Kalman filter estimation is not rooted in any analysis of contango/backwardation in the term structure of VIX futures or considerations of changes in the volatility of VIX; it only assesses the effects of such changes on α and β. With its random-walk assumptions, there is some persistence in the estimates that it generates. Still, it is able to adjust to changes in the market environment relatively quickly.

Exhibit 10 summarizes the performance statistics for the four EMN portfolios. All four EMN portfolios have low correlations with the S&P 500 Index, and their betas in the traditional CAPM mode are approximately zero,

which qualifies them as EMN portfolios. In addition, the Kalman filter portfolio has superior performance: It has the best total return and the best Sharpe ratio. In addition, its correlation with the S&P 500 Index is very low. We think this superior performance comes from the Kalman filter's ability to identify the current market structure.

Looking again at our method for choosing weights, we want the portfolio to be market neutral with respect to the S&P 500 so that its forecasted beta is zero. However, in reality the current beta is the practical forecast of the future beta; we therefore want the most up-to-date estimates of beta in order to assign the weights. There could be two sets of weights with opposite signs that

63- and 126-Day Rolling-Window EMN Portfolios Using VIX Short-Term and Mid-Term Futures Indexes (March 22, 2006–June 30, 2015)


Panel A: EMN Portfolios

Panel B: Drawdowns
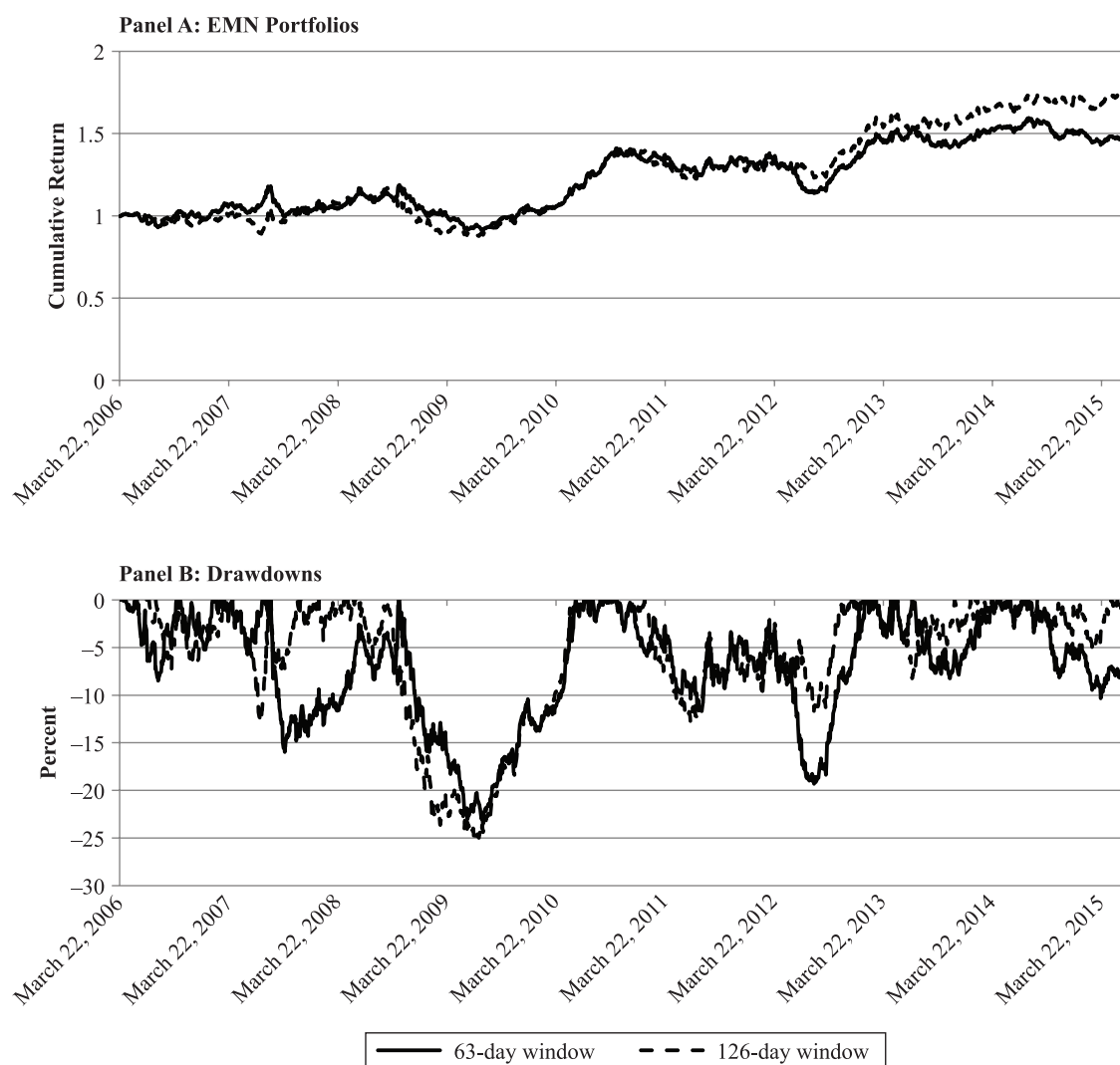
achieve beta neutrality. If the market were efficient, either choice would have an alpha of zero. However, because of leverage constraints such as what we saw in FP, there are arbitrage opportunities to achieve positive alpha with the proper selection of the weights. Our static OLS results demonstrate that a profitable trade exists, although the weights do not adjust to changing market conditions. Rolling regression would give us estimates of weights that adjust over time, but we have argued that these estimates are subject to several drawbacks. The goal is to obtain estimates that adjust quickly and reliably to changing market conditions, and we find that Kalman

filter estimation provides such estimates and indeed improves on the simpler rolling regression system.

Exhibit 11 presents the daily return statistics of the Kalman filter EMN portfolio in the form of histograms. As seen in the exhibit, the Kalman filter EMN portfolio has a very narrow distribution in spite of the effects of the financial crisis and the subsequent bull market. Due to its construction methods, maximum daily swings are less than 3%. Compared with the S&P 500 Index, VIX Short–Term Futures Index, and VIX Mid–Term Futures Index, the Kalman filter EMN portfolio has negligible skewness. The Kalman filter strategy is very

Kalman Filter EMN Portfolio Using VIX Short-Term and Mid-Term Indexes (March 1, 2006–June 30, 2015)

Panel A: Kalman Filter EMN Portfolio

Panel B: Drawdowns
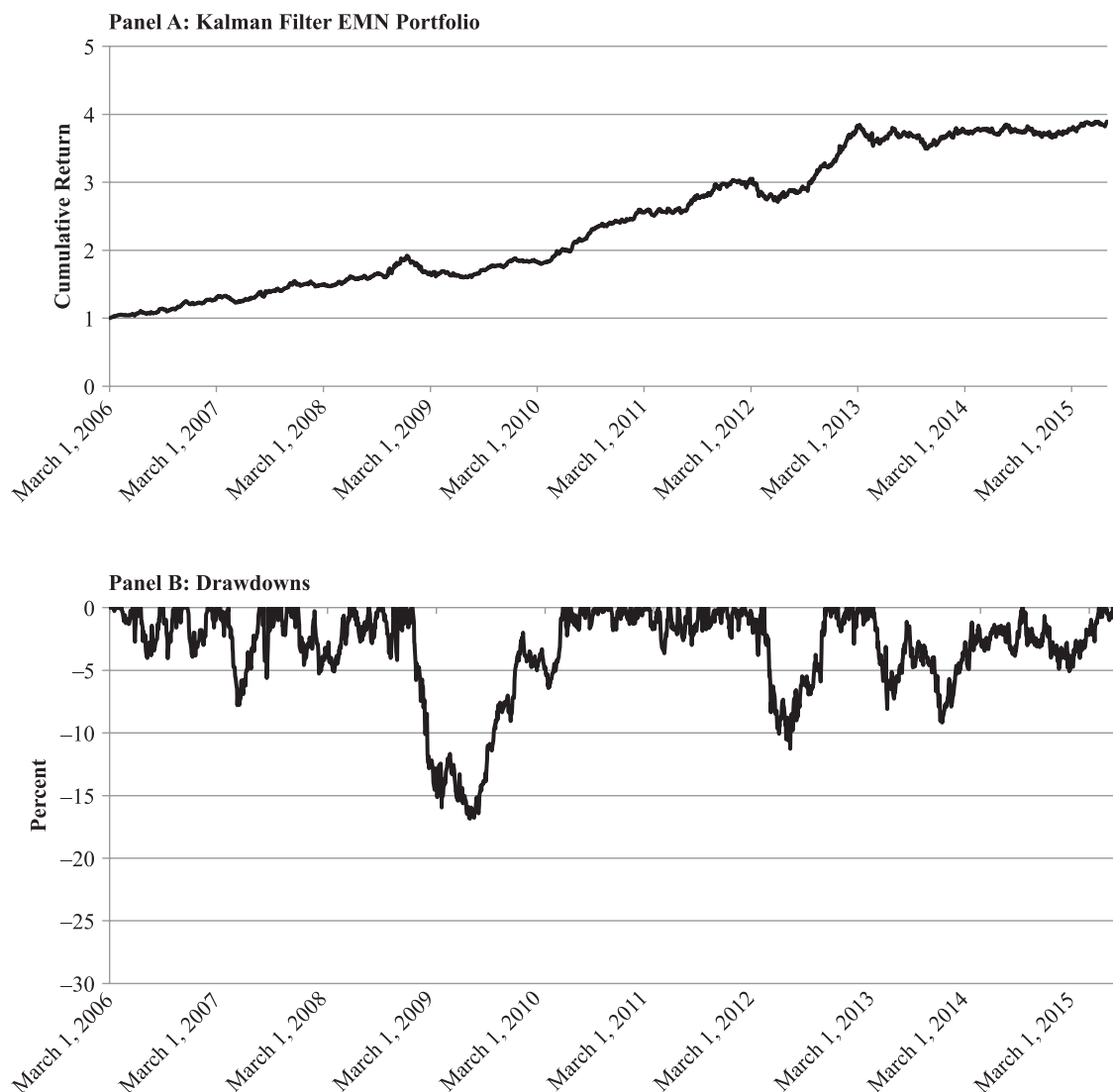
different from other EMN strategies: The daily returns are highly symmetric, with a slightly positive mean reflecting its positive alpha. This gives it the potential to make money regardless of underlying market movements.

We evaluated the performance of the strategy at weekly, monthly, quarterly, and yearly intervals. We define a winning (losing) trade to be a trade that yields a positive (negative) cumulative daily return in the given time interval. Exhibit 12 shows the results of daily trades at the alternative time intervals. In the exhibit, the win/loss ratio rises as the time interval increases. At longer horizons with a symmetric distribution, the law of large

numbers generates a good return profile despite the fact that beta is approximately zero.

Hedge Fund Research Inc. (HFR) provides data on hedge fund performance segregated into indexes according to fund strategy; a hedge fund that reports to HFR declares its strategy and is sorted into the appropriate index. We examine two HFR indexes: (1) the HFRI Equity Market Neutral Index, which we refer to as HFR EMN, as representative of a class of market-neutral hedge funds; and (2) the HFRX Relative Value Volatility Index, which we refer to as HFR Vol, as representative of a class of hedge funds engaged in

Performance Summary Statistics of the Alternative Approaches for Constructing EMN Portfolios (June 21, 2006–June 30, 2015)

Statistics	Static	OLS-126	OLS-63	Kalman Filter
Annual mean (%)	6.01	6.03	5.07	15.30
Volatility (%)	9.25	9.13	9.02	9.09
Sharpe ratio	0.53	0.53	0.44	1.46
Skewness	−0.31	0.16	−0.07	−0.09
Excess kurtosis	5.07	4.07	2.32	2.21
Total return (%)	69.21	69.47	56.09	260.50
Max drawdown (%)	25.37	25.24	23.53	16.84
Correlation with S&P 500	0.02	0.02	−0.01	−0.06
Correlation with SPVXSP	−0.23	−0.02	0.03	0.07
Correlation with SPVXMP	0.21	0.03	0.06	0.08
α (daily %)	0.02	0.02	0.02	0.06
β	0.01	0.01	0.00	−0.02

volatility trading. In Exhibit 13, we show the correlation matrix for the S&P 500 Index, the two HFR indexes, and our Kalman filter VIX EMN portfolio.

The large positive correlation of the HFRI Equity Market Neutral Index with the S&P 500 Index reveals that funds declaring themselves as EMN are not always market neutral. In fact, the correlation of the HFRX Relative Value Volatility Index and the S&P 500 Index is smaller. However, because the Kalman filter VIX EMN portfolio dynamically hedges, it has very low correlation with the S&P 500 Index and the two HFR indexes. Overall, in the period covering the financial crisis and subsequent bull market, the Kalman filter EMN portfolio delivers low correlation with the broad market, low volatility, and good performance.

ROBUSTNESS CHECKS

Robustness and Randomization of Parameters

The Kalman filter parameters are estimated by MLE using historical data. The estimations could vary using different optimization techniques, which raises the question of whether the methodology yields the best parameters for purposes of portfolio construction. To examine this issue, we introduce randomness in the Kalman filter parameters. We allow all six estimated parameters in the Q and R matrixes in the Kalman filter equations to vary

independently and randomly within two standard deviations of the MLE estimated values. Then we recompute the EMN portfolio with the new sets of noisy parameters. This exercise is repeated 100 times. Exhibit 14 shows 0%, 25%, 50%, 75%, and 100% performance quartiles of 100 simulations of five representative portfolios. Here, we see strong similarities in portfolio performance despite the randomness in the parameters.

Exhibit 15 shows the results in the form of a histogram of the final portfolio values. The final portfolios each show positive returns, a result that further corroborates the robustness of the algorithm. The robustness stems from the way the Kalman filter controls the drift of the underlying random walk. Even with randomized changes in the presumed drift rate, the computation of α and β is maintained with sufficient precision to achieve good EMN portfolio performance.

Robustness and the Effects of Intraday Price Volatility

As another robustness check, we note that the portfolio construction method is based on the assumption that computation of the weights $w_{1,t}$ and $w_{2,t}$ takes place at the close of the market. However, it may not be possible to compute the ideal weights at the close. This would be expressed as deviations in the portfolio weights $w_{1,t}$ and $w_{2,t}$ from their targeted values. Thus, to capture this effect, we introduce randomness into the daily portfolio weights $w_{1,t}$ and $w_{2,t}$. We allow the weights to vary independently and randomly each day within $\pm \gamma$ percent of the values generated by our computation method, where γ is set alternately at 1, 3, and 5 percent. For each γ, we recompute the EMN portfolios 100 times with the sets of randomized weights. With the weights subjected to noise, gross leverage $\left| w_{1,t} \right| + \left| w_{2,t} \right|$ could be slightly greater or less than 1.

Exhibit 16 shows the performance outcomes in the form of histograms. Here, we see that the dispersions of the histograms increase as γ increases. However, for each γ, performance is always positive, and the means of the histograms are approximately the same.

Robustness Using Exchange-Traded Notes

As a further check on the trading strategy, we replace the VIX futures indexes with their respective exchange-traded notes (ETNs). That is, we replace SPVXSP with

Exhibit 11

Histograms of Daily Returns for the Kalman Filter EMN Portfolio, S&P 500 Index, and VIX Short-Term and Mid-Term Indexes (March 1, 2006–June 30, 2015)
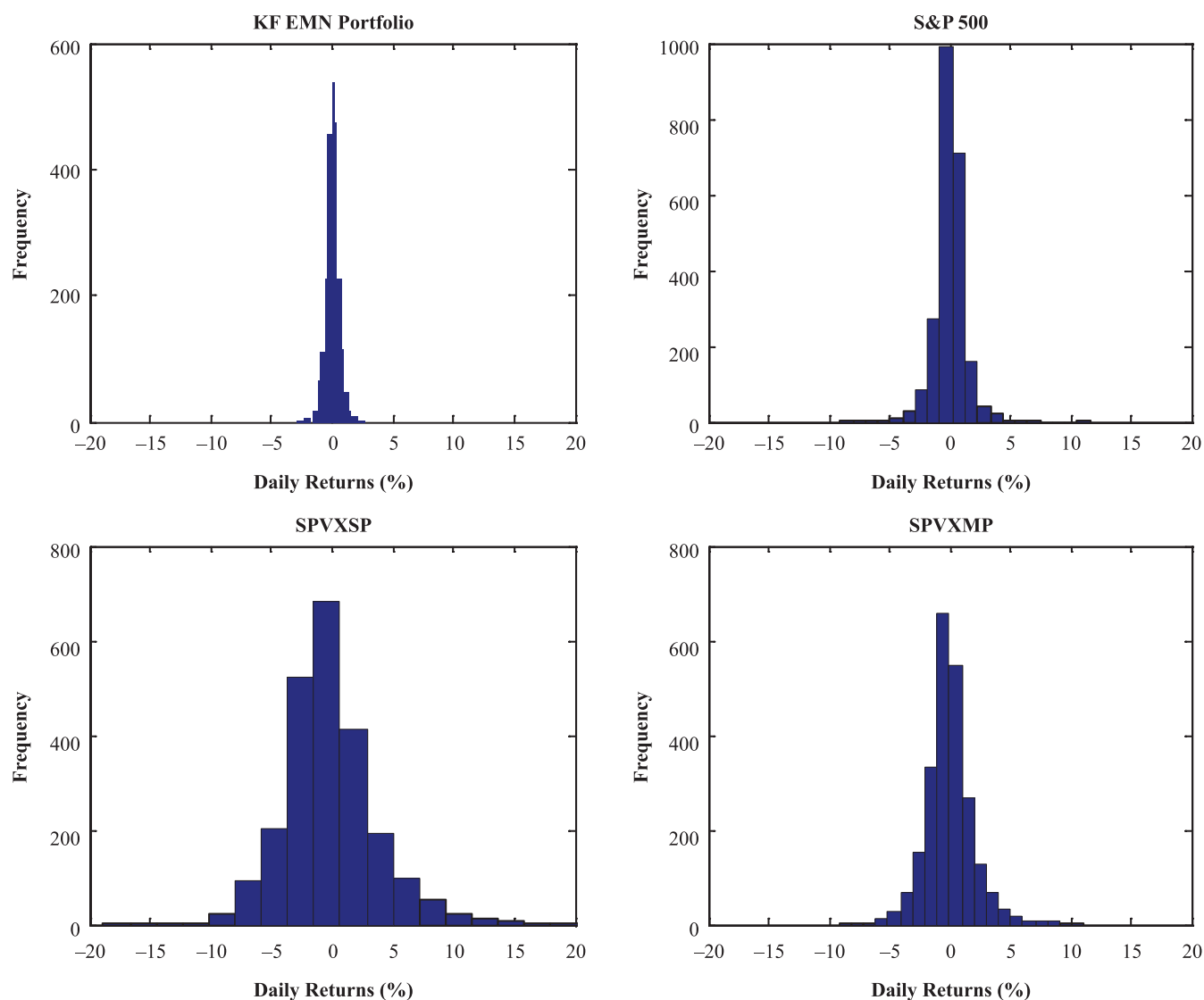
Exhibit 12

Wins and Losses at Weekly, Monthly, Quarterly, and Yearly Intervals (April 3, 2006–June 30, 2015)

Time Interval	Number of Wins	Number of Losses	Ratio
Daily	1,308	1,040	1.26
Weekly	295	193	1.53
Monthly	77	35	2.20
Quarterly	28	10	2.80
Yearly	9	1	9.00

VXX, and SPVXMP with VXZ. These ETNs track the underlying indexes very well and have gained popularity since their inception in early 2009. We compute results using the daily closing prices of the ETNs.

Exhibit 17 shows results comparing the cumulative performance of the ETN portfolio and the portfolio constructed with the original VIX futures indexes. The starting date in the exhibit differs from that in earlier exhibits because ETNs were issued after the creation of the VIX indexes. The two portfolios exhibit similar

growth paths over more than six years. The discrepancy between the cumulative performances of the two portfolios could be a result of (1) tracking error of the ETNs with respect to the underlying indexes, (2) the operating expenses charged by the ETN provider, and (3) the extension of futures trading hours beyond the equity market close.

Robustness with Trade Avoidance and Transaction Costs

As yet another robustness check, we note that hedge funds may seek to avoid trades where day-to-day changes in computed weights are small. Exhibit 18 shows that the Kalman filter–based construction method can produce rapid changes in the weights $w_{1,t}$ and $w_{2,t}$. We note in passing that the change of signs of the weights seen in the exhibit is a direct challenge to the FP method, where the high beta stocks are always sold short. This suggests that, although the FP method could be profitable over a very long horizon, performance using FP could suffer in the short to medium term. In any event, using our approach, the weights tend to be little changed for days at a time, which opens the possibility that small changes in the weights in such periods could be ignored, reducing trading activity without appreciably sacrificing performance.

To examine this issue, we note that because the absolute values of the weights sum to 1, where one weight deviates in fixed absolute amount D, so does the other weight; we compute performance where the weights are not adjusted until each differs absolutely by more than D from the last established weight. Exhibit 19 shows the relationship between values of D and the final portfolio value. Here, we see that the imposition of the trading threshold does not generally impair performance. As D increases, more and more trades—those involving relatively small changes in the weights—are

E X H I B I T 1 3

Correlation Matrix for the Monthly Returns of the S&P 500 Index, HFRX Relative Value Volatility Index, HFRI Equity Market Neutral Index, and Kalman Filter VIX EMN Portfolio (April 30, 2006–June 30, 2015)
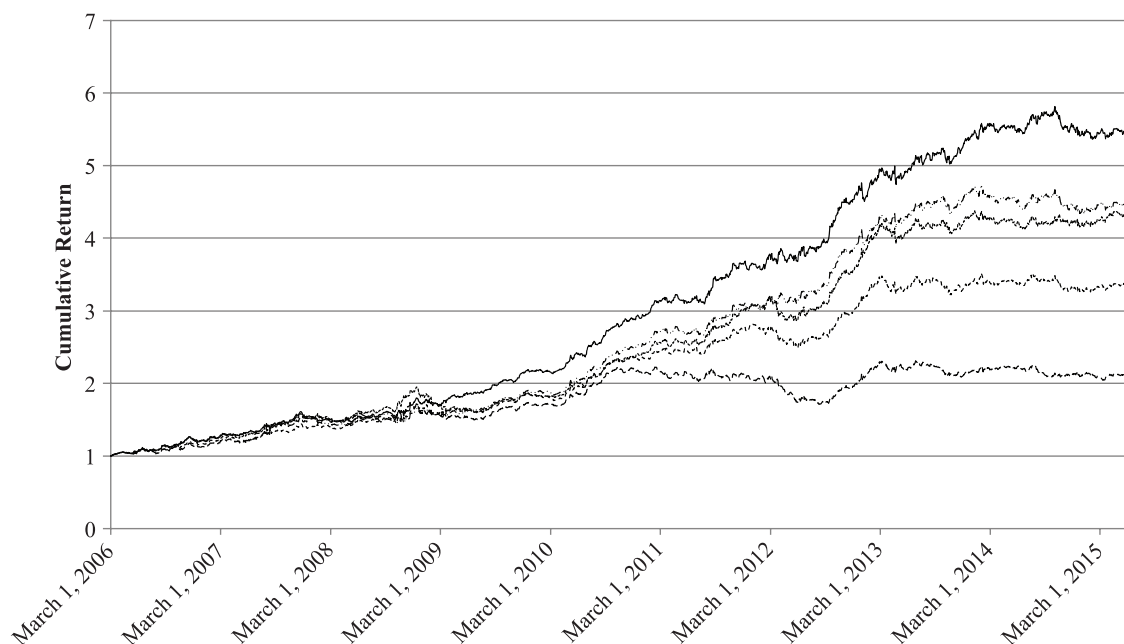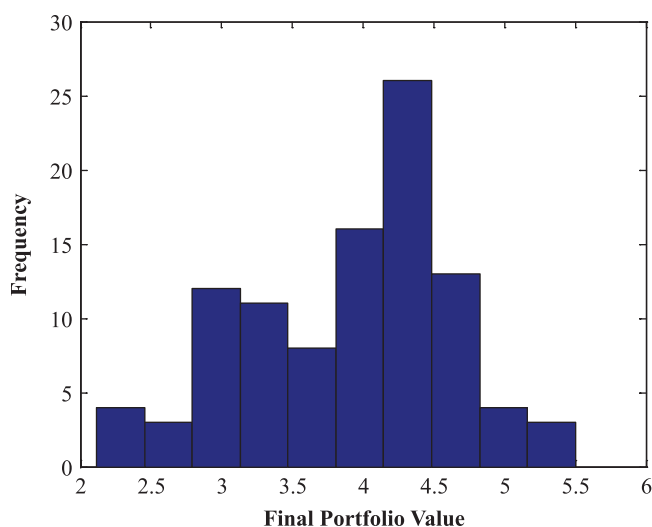
	S&P 500	HFRX Vol	HFRI EMN	VIX EMN
S&P 500	1.00	0.15	0.52	−0.05
HFR Vol	0.15	1.00	0.32	−0.04
HFR EMN	0.52	0.32	1.00	0.04
VIX EMN	−0.05	−0.04	0.04	1.00

E X H I B I T 1 4

Performance for Five Portfolios Using Randomized Kalman Filter Parameters (March 1, 2006–June 30, 2015)

Histogram of Final Portfolio Value Using Randomized Parameters

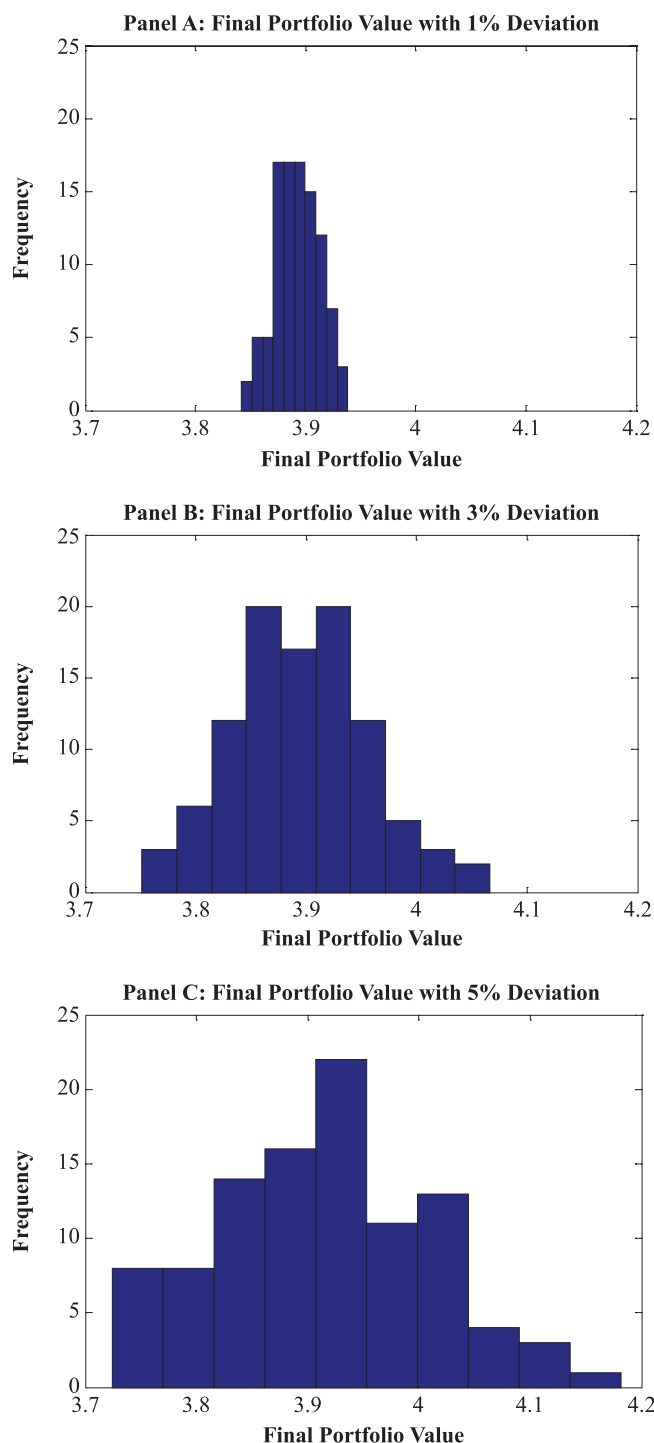**Histograms of Final Portfolio Values Using Randomized Weights**

eliminated. The trades involving relatively large changes in weights are not eliminated, and these trades drive performance.

As a final check on our strategy, we incorporate transaction costs into our analysis of trade avoidance. Here, we use data from the ETNs for the underlying VIX futures indexes to incorporate estimates of trading costs associated with passive rolling from contract to contract. Based on bid–ask spreads and online broker commissions, we assume per-trade costs of 0.037% for the VXX and 0.087% for VXZ for an investor. The annual expense ratio is 0.89% for both the VXX and VXZ ETNs, which translates to a 0.0035% cost per day simply to maintain the positions. On most days, only a small fraction of each security needs to be bought or sold. Therefore, the daily cost is the sum of the 0.0035% rolling cost and the actual trading costs multiplied by the changes in weights. Exhibit 20 shows the results using these cost estimates.

Naturally, overall performance goes down where trading costs are applied. Still, overall performance is robust, and the advantage of reduced–rebalance schemes becomes more obvious with trading costs included in the calculation. Performances actually improve for all reduced–rebalance cases when compared to the daily rebalance base scheme.

Robustness Using Alternative VIX Indexes

The question arises as to whether the performance of the Kalman filter method applied to the two VIX futures indexes is simply fortuitous. Put another way, are there deep connections among all the VIX futures indexes? Using the Kalman filter algorithms, we construct four additional EMN portfolios that combine the VIX Short-Term Futures Index and, one by one, the four other VIX futures indexes. Exhibit 21 presents the results.

Exhibit 22 summarizes performance results for the original portfolio and the four additional portfolios. All five portfolios have suppressed volatility and betas of
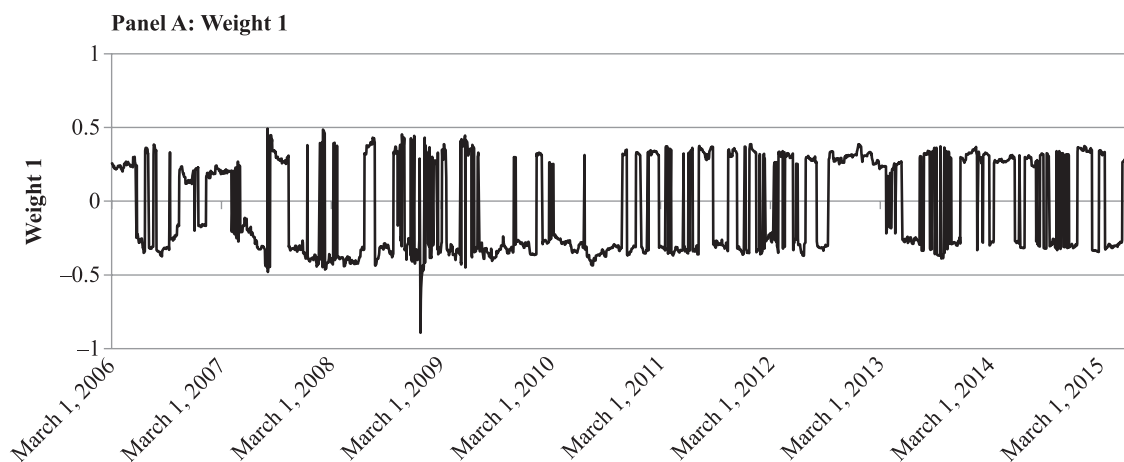
E X H I B I T 17
Cumulative Returns for the Kalman Filter EMN Portfolio Using ETNs and the Original Index Values (April 29, 2009–June 30, 2015)
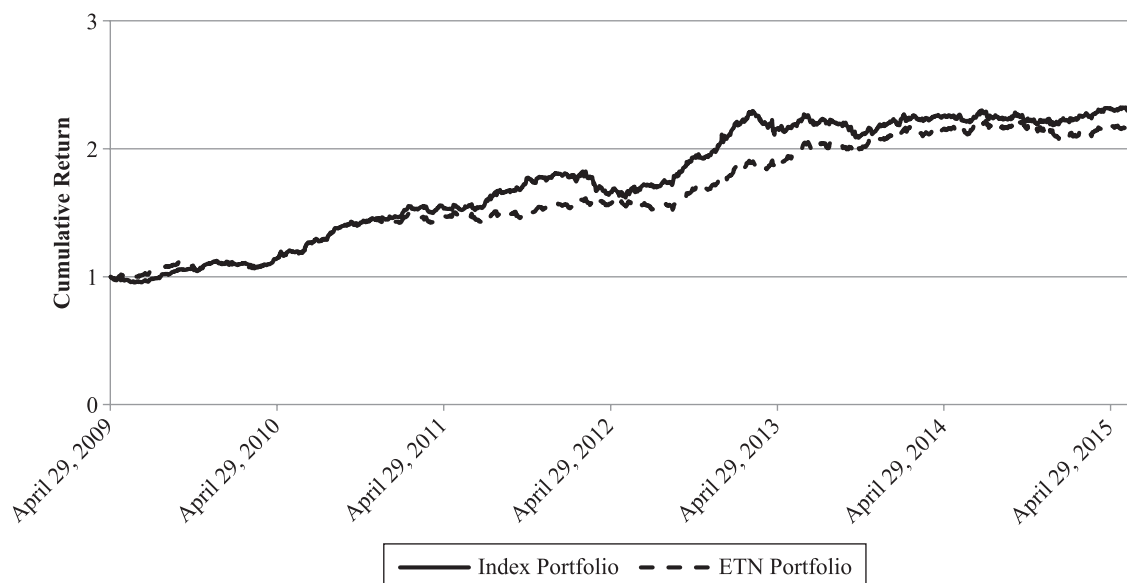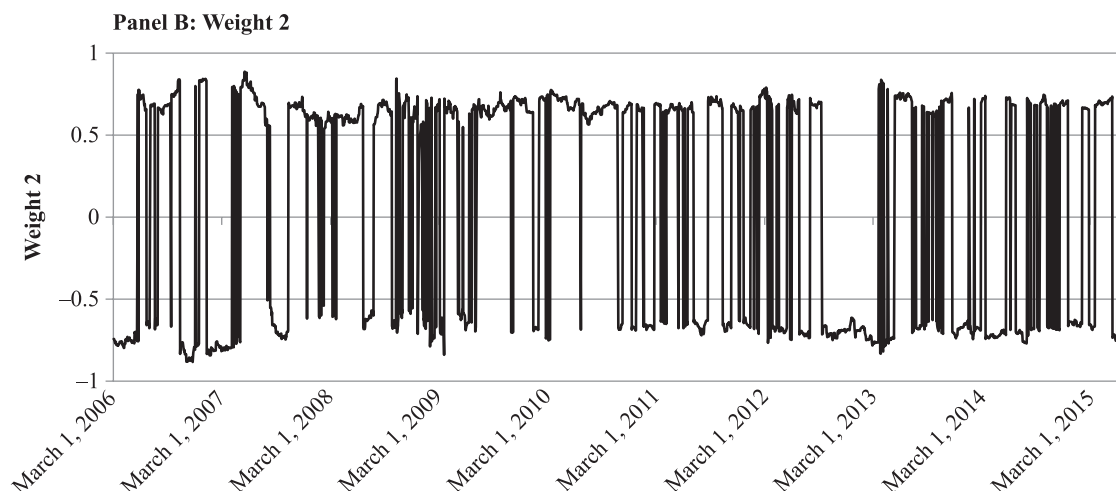

E X H I B I T 18
Daily Portfolio Weights (March 1, 2006–June 30, 2015)

Panel A: Weight 1

(*continued*)

Panel B: Weight 2

E X H I B I T **19**

Final Portfolio Value Where Trades Are Subject to Threshold *D*

D	Trades	Final Portfolio Value	Maximum Drawdown
0.00	2,349	3.89	16.84
0.05	371	3.96	16.17
0.10	250	4.08	16.32
0.15	225	3.67	16.73
0.20	214	3.82	15.34
0.25	211	3.96	14.26
0.30	210	4.35	14.26

E X H I B I T **20**

Final Portfolio Value Where Trades Are Subject to Threshold *D* with the Assumed Transaction Costs

D	Trades	Final Portfolio Value	Maximum Drawdown
0.00	2,349	2.58	20.40
0.05	371	2.87	19.42
0.10	250	2.99	19.23
0.15	225	2.70	19.58
0.20	214	2.82	18.19
0.25	211	2.92	16.87
0.30	210	3.21	16.87

almost zero. With the exception of the portfolio containing SPVIX2ME, all portfolios have good Sharpe ratios and superior return characteristics. Goetzmann et al. [2004] showed that the Sharpe ratio can be misleading where return distributions are negatively skewed. This is not a concern because our EMN portfolios have negligible skewness. The results in the exhibit seem to indicate that combining the VIX Short-Term Futures Index with the relatively longer maturities may provide better hedging capability in terms of portfolio skewness and kurtosis.

CONCLUSION

VIX instruments have gained in popularity among market participants, but their large swings pose risks. In this article, we propose using the Kalman filter to hedge risks dynamically. Unlike conventional approaches, such as static OLS or rolling regression, the Kalman filter responds more promptly to changes in market conditions. This strategy allows the user to adjust hedging ratios more rapidly, thus reducing portfolio volatility. The fast response from the Kalman filter opens a way to compute the equivalent Greeks for complicated VIX derivatives, which could allow the use of novel VIX derivatives for risk-management purposes. While maintaining a portfolio with almost no market exposure, our approach generates positive alpha, which in essence means that the dynamic system is able to buy the winner and sell short the loser.

Traditional EMN strategies are based on the concept that a pricing relationship that appears to have diverged from the perceived normal relationship will converge to that relationship. However, the divergence can widen during periods of market stress. In contrast, because our approach does not bet on convergence, it may avoid the

Performance Results for Kalman Filter EMN Portfolios Using the VIX Short-Term Futures Index Combined with Alternative VIX Futures Indexes (March 1, 2006–June 30, 2015)
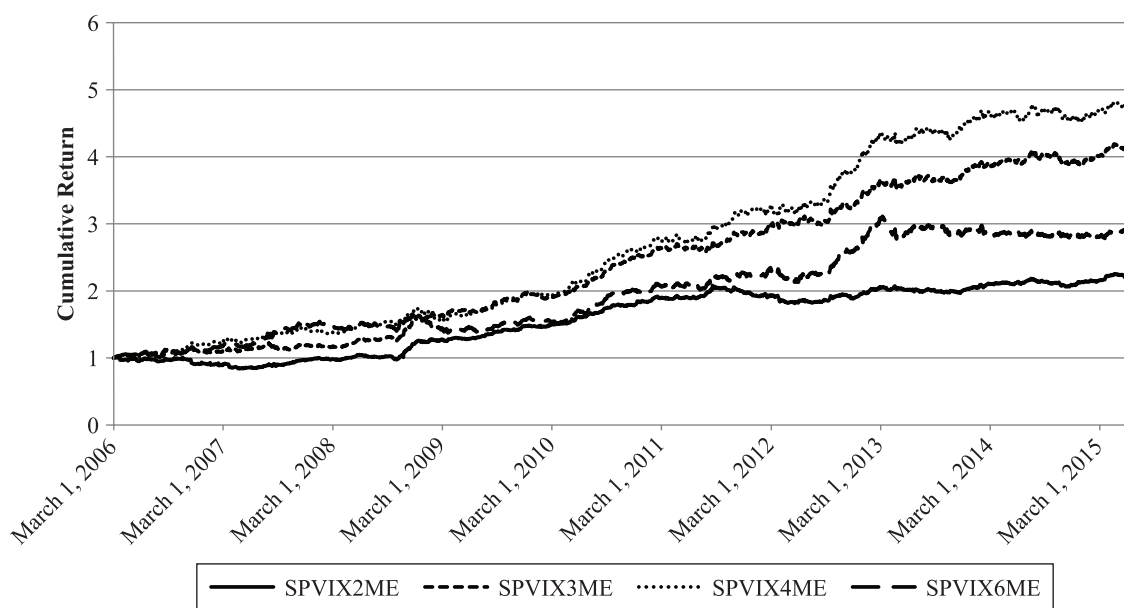
Summary of Performance Results with Alternative VIX Futures Indexes

Statistics	SPVIX2ME	SPVIX3ME	SPVIX4ME	SPVXMP	SPVIX6ME
Annual mean (%)	8.73	16.42	18.27	15.70	11.95
Volatility (%)	7.64	8.02	8.67	9.06	9.81
Sharpe ratio	0.95	1.76	1.81	1.49	1.04
Skewness	−0.45	0.15	0.04	−0.09	−0.08
Excess kurtosis	12.79	4.70	2.14	2.23	2.77
Total return (%)	118.12	312.40	377.90	289.36	186.44
Max drawdown (%)	16.58	8.93	11.47	16.84	19.15
Correlation with S&P 500	−0.04	−0.01	−0.03	−0.06	−0.09
α (daily %)	0.03	0.06	0.06	0.06	0.04
β	−0.01	0.00	−0.01	−0.02	−0.04

substantial losses that often occur in traditional strategies. It may be possible to extend the Kalman filter EMN portfolio concept to multi–asset portfolios and to other highly correlated and volatile markets.

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