

# Can Structural Models Price Default Risk? Evidence from Bond and Credit Derivative Markets\*

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## Abstract

Using a set of structural models, we evaluate the price of default protection for a sample of US corporations. In contrast to previous evidence from corporate bond data, CDS premia are not systematically underestimated. In fact, one of our studied models has little difficulty on average in predicting their level. For robustness, we perform the same exercise for bond spreads by the same issuers on the same trading date. As expected, bond spreads relative to the Treasury curve are systematically underestimated. This is not the case when the swap curve is used as a benchmark, suggesting that previously documented underestimation results may be sensitive to the choice of risk free rate.

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# 1 Introduction

A widespread view amongst financial economists is that structural models of credit risk following Black and Scholes (1973) and Merton (1974), although theoretically appealing, underestimate the actual default risk discount on credit risky securities. Several studies from the 1980's onwards document the models producing credit spreads lower than actual corporate bond spreads.<sup>1</sup> A recent financial innovation, the credit default swap (CDS) permits the measurement of the default risk component of a corporate issuer in isolation. In addition, CDS are commonly thought to be less influenced by non-default factors.<sup>2</sup> The CDS market thus provides an interesting alternative source of data to reassess the empirical performance of structural models that by construction only measure default risk.

For a selection of structural models, we compare predicted levels of credit default swap (CDS) premia with their market counterparts. In contrast to what has previously found on corporate bond data, CDS premia are not systematically underestimated. In fact, one of our studied models has little difficulty on average in predicting their level.

For robustness, we also compare the models' theoretical bond spreads to their market counterparts. CDS contracts are closely related to corporate bonds by an arbitrage argument. A package of a par floating rate corporate bond and protection bought with a CDS is a risk free investment. Given that a CDS contract requires no up front payment (i.e. it is unfunded), the comparison with the bond requires taking into account the funding rate of a bondholder. In practice, this implies that the relevant measure of the bond yield spread that is comparable to the CDS price is the spread to the swap curve.<sup>3</sup> In contrast most previous empirical research on corporate bonds use the Treasury curve as a benchmark. As a result we carry out our analysis using reference yield curves based both on Treasury yields and interest swap rates.

We use a simple building block approach to develop a pricing formula for credit default swaps (henceforth, CDS). The formula is applied in the framework of three distinct structural models: Leland (1994), Leland and Toft (1996) and Fan and Sundaresan (2000). By implementing multiple models, we hope to gauge the robustness of our results to specific model assumptions. An important difference between our approach and that of Longstaff, Mithal, and Neis (2004) is that we do not rely on market prices of corporate bonds or CDS as inputs to our estimation. Thus, we need not assume that CDS prices are driven solely by default risk. Instead, we estimate the

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<sup>1</sup>See Jones, Mason, and Rosenfeld (1984), Jones, Mason, and Rosenfeld (1985), Ogden (1987) and Lyden and Saranati (2000).

<sup>2</sup>A recent study of non-default components of CDS premia can be found in Tangy and Yan (2006).

<sup>3</sup>A more detailed explanation is provided below.

structural models using firm-specific balance sheet and market data on stock prices, and then compute the term structures of risk-adjusted default probabilities and the corresponding prices for corporate bonds and CDS for the same corporate issuer.<sup>4</sup> By using pairs of contemporaneous transactions and quotes for default swaps and a bond issued by the reference entity, we can compare the estimated bond yield spreads and CDS spreads with two separate sets of data.

Consistent with previous evidence, the models do systematically underpredict bond spreads when benchmarked against Treasuries. This is however not the case for spreads computed against the swap curve. The results based on the swap curve are very similar to the those for default swaps. This suggests that the documented underestimation of bond spreads may to a large degree be attributable to the choice of benchmark risk free curve.

In more detail, we find that our three models tend to systematically underestimate bond spreads and, more importantly, that this is not the case for CDS premia when Constant Maturity Treasury rates are being used as riskfree benchmark. For our base case specifications, the Leland (1994) and Fan and Sundaresan (2000) models underestimate bond spreads by 91 and 67 basis points respectively. The Leland and Toft (1996) model underestimates much less – by ca. 59 basis points. These numbers are not dissimilar to previous findings by, for example, Eom, Helwege, and Huang (2004). When we consider CDS premia, a very different picture emerges. On comparison of model and market CDS premia we find that the Leland (1994) and Fan and Sundaresan (2000) models underestimate CDS premia by 43 and 19 basis points, much less so than for bonds, in particular for the latter. The benchmark implementation of the Leland and Toft (1996) actually underestimates CDS premia by, on average, 2 basis points. However, when swap rates are used as benchmark, the systematic underestimation by the structural models is not present. The LT model underestimates mean bond spreads by 2% only, while the underestimation by the L and FS models is significantly reduced.

In an additional effort to understand the model’s performance, we relate their residuals by means of linear regressions to default risk and non-default proxies. We find little evidence of any default risk component in either bond or CDS residuals. For example, credit ratings are unable to explain residual spread levels. A variable measuring the difference between bond yield indices of different credit ratings does have explanatory power, however. This may suggest that our implementation methodology is not properly accounting for risk premia. It could also be a

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<sup>4</sup>We use a maximum likelihood technique developed by Duan (1994) and evaluated by Ericsson and Reneby (2005). The latter show by means of simulation experiments that the efficiency of this method is superior to the more common approach used in previous empirical studies on the use of structural models for valuing credit risky securities. See for example, Jones, Mason, and Rosenfeld (1984), Ronn and Verma (1986) and Hull (2000).

result of non-default components present in those indices. In the residuals for bonds, we find evidence of an illiquidity component of between 10 and 20 basis points. CDS residuals reveal no such component.

Taken together with our results on levels of bond spreads and CDS premia, this is consistent with reasonably specified structural models being able to capture the default risk priced in bond and credit derivative markets. Past results on the underestimation of bond spreads may be an effect of ignoring the funding cost of bond investors.

The remainder of this paper is structured as follows. The following Section introduces the models and Section 3 the empirical methodology. Thenceforth, section 4 describes both the bond and CDS datasets. Section 5 draws out the implications of the results and finally, section 6 concludes.

## 2 Setup

Here we initially present the three structural models that we will study. The ensuing description is brief and we refer the reader to the original papers for details. These do not, however, treat the valuation of credit default swaps. Therefore, we utilize a simple building block approach to demonstrate how to value a CDS and its reference bond.

### 2.1 The Models

Consider first the common characteristics of the three structural models: Leland (1994), Leland and Toft (1996) and Fan and Sundaresan (2000). The fundamental variable in all models is the value of the firm's assets (the unlevered firm), which is assumed to evolve as a geometric Brownian motion under the risk-adjusted measure:

$$d\omega_t = (r - \beta)\omega_t dt + \sigma\omega_t dW_t$$

The constant risk-free interest rate is denoted  $r$ ,  $\beta$  is the payout ratio,  $\sigma$  is the volatility of the asset value and  $W_t$  is a standard Wiener-process under the risk-adjusted measure.

Default is triggered by the shareholders' endogenous decision to stop servicing debt. The exact asset value at which this occurs is determined by several parameters as well as the characteristics of the respective models, but is always a constant which we denote by  $L$ .

The value of the firm differs from the value of the assets by the values of the tax shield and

the expected bankruptcy costs. Coupon payments are tax deductible at a rate  $\tau$  and the realized costs of financial distress amount to a fraction  $\alpha$  of the value of the assets in default (i.e.  $L$ ). In this setting, the value of the firm ( $\mathcal{F}$ ) is equal to the value of assets *plus* the tax shield ( $\mathcal{TS}$ ) *less* the costs of financial distress ( $\mathcal{BK}$ ). The value of the firm is, of course, split between equity ( $\mathcal{E}$ ) and debtholders ( $\mathcal{D}$ ) and thus all models share the following basic equalities:

$$\begin{aligned}\mathcal{F}(\omega_t) &= \omega_t + \mathcal{TS}(\omega_t) - \mathcal{BK}(\omega_t) \\ &= \mathcal{E}(\omega_t) + \mathcal{D}(\omega_t)\end{aligned}\tag{1}$$

Note that the formulae for the components depend on the model, but that they are all independent of time.

In the interest of brevity, the formulae are relegated to the appendix, and this paper merely discusses differences between the applied models. First, Leland (1994) is a natural benchmark model where debt is perpetual and promises a continuous coupon stream  $C$ . Financial distress triggers immediate liquidation and no renegotiation is possible.

In Leland and Toft (1996), the firm continuously issues debt of maturity  $\Upsilon$ ; therefore, the firm also continuously redeems debt issued many years ago. Hence, at any given time, the firm has many overlapping debt contracts outstanding, each serviced by a continuous coupon. Coupons to individual debt contracts are designed such that the total cash flow to debt holders (the sum of coupons to all debt contracts plus nominal repayment) is constant. Letting  $\Upsilon \rightarrow \infty$ , the model converges to the Leland model. For shorter maturities, the need to redeem debt places a higher burden on the firm's cash flows. Consequently, the default barrier in the Leland & Toft model tends to be much higher than in the Leland model for short and intermediate debt maturities.

In Fan and Sundaresan (2000) debt is, as in Leland, single-layered and perpetual but creditors and shareholders can renegotiate in distress to avoid inefficient liquidations. Consequently, the default barrier in the former model is typically lower than in its Leland counterpart. To the extent that equity holders can service debt strategically, bondholder bargaining power is captured by the parameter  $\eta$ . If the bargaining power is nil, no strategic debt service takes place and the model converges to the Leland model.

## 2.2 Valuing the bond

Next think about what kind of real world ‘debt’ the authors had in mind when building the three structural models above. A firm’s debt consists of bank loans, bonds, accounts payable, salaries due, accrued taxes etc. Dues to suppliers, employees and the government are substitutes for other forms of debt. Part of the price of a supplied good and part of salary paid can be viewed as corresponding to compensation for the debt that, in substance, it constitutes. The cost of debt consequently includes not only regular interest payments to lenders and coupons to bondholders, but also fractions of most other payments made by a company. Clearly, a comprehensive model of all these payments would not be tractable and we think of the three models as portraying firms’ aggregate debt rather than a particular bond issue – such as the reference obligation of the CDS.

However, we do need a pricing formula which also accounts for the reference obligation – for robustness, we will let the models price both the CDS and the corresponding bond in order to investigate whether the overestimation of the spread is indeed smaller in the former case. To this end, we apply a bond pricing model that takes discrete coupons, nominal repayment and default recovery into account.<sup>5</sup> To express the value of the bond we make use of two building blocks, a binary option  $H(\omega_t, t; S)$  and a dollar-in-default claim  $G(\omega_t, t; S)$ . The former pays off \$1 at maturity  $S$  if the firm has not defaulted before that, the latter pays off \$1 upon default should this occur before  $S$ ; the value of both depend upon the firms asset value  $\omega_t$  and current time  $t$ . The formulae for the binary option and the dollar-in-default claim are, for a given default barrier  $L$ , identical in all three structural models.

**Proposition 1** *A straight coupon bond. The value of a coupon bond with  $M$  coupons  $c$  paid out at times  $\{t_i : i = 1..M\}$  is*

$$\begin{aligned} \mathcal{B}(\omega_t, t) &= \sum_{i=1}^{M-1} c \cdot H(\omega_t, t; t_i) \\ &\quad + (c + P) \cdot H(\omega_t, t; T) \\ &\quad + \psi P \cdot G(\omega_t, t; T) \end{aligned}$$

*The formulae for  $H$  and  $G$  are given in the appendix.*

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<sup>5</sup>This bond pricing model was used in Ericsson and Reneby (2004) and was shown to compare well to reduced form bond pricing models.

The value of the bond is equal to the value of the coupons ( $c$ ), the value of the nominal repayment ( $P$ ) plus the value of the recovery in a default ( $\psi P$ ). Each payment is weighted with a claim capturing the value of receiving \$1 at the respective date.

Note that the above formula for the reference bond is not directly related to the debt structure of the firm. Specifically, coupon payments to the bond are unaffected by the strategic debt service in the Fan & Sundaresan model, and by the debt redemption schedule elaborated in the Leland & Toft model. The choice of model affects the bond formula solely via the default barrier  $L$ .

## 2.3 Valuing the CDS

A CDS provides insurance for a specified corporate bond termed the *reference obligation*. The firm issuing this bond is designated the *reference entity*. The seller of insurance, the *protection seller*, promises, should a default event occur, to buy the reference obligation from the *protection buyer* at par.<sup>6</sup> The credit events triggering the CDS are specified in the contract and typically range from failure to pay interest to formal bankruptcy. For the CDS, the protection buyer pays a periodic fee rather than an up-front price to the seller. When and if a credit event occurs (at time  $\mathcal{T}$ ), the buyer is also required to pay the fee accrued since the previous payment. Hence fee payments, although due at discrete intervals, fit nicely into a continuous modelling framework. Note also that there is no requirement that the protection buyer actually own the reference obligation, in which case the CDS is used for speculation rather than protection.

The valuation of a CDS thus involves two parts, the premium paid by the protection buyer and the potential buy-back by the protection seller. Letting  $T^*$  denote maturity of the CDS and  $Q$  the fee, the value of the premium at time  $t$  is

$$E' \left[ \int_t^{T^*} e^{-r(s-t)} \cdot Q \cdot I_{\mathcal{T} \not\leq s} ds \right]$$

where we let  $I_{\mathcal{T} \not\leq s}$  be the indicator function for nondefault before  $s$  and  $E'$  denotes the expectations under the standard pricing measure. The maturity of the credit default swap is typically shorter than the maturity of the reference obligation ( $T$ ). In fact, the by far most common maturity in practice is  $T^* = 5$  years.

Assume that a bond holder in the event of bankruptcy recovers a fraction  $\psi$  of par,  $P$ . The second part of the value of a CDS therefore is the expected value of receiving, upon default of the

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<sup>6</sup>In practice, there may be cash settlement or delivery of another (non-defaulted) bond in place of direct purchase but we refrain from that complication here.

firm, the difference between the bond's face value and its market price,  $P - \psi P$ :

$$E' \left[ e^{-r(\mathcal{T}-t)} \cdot (P - \psi P) \cdot I_{\mathcal{T} < T^*} \right]$$

The expectation is conditional on default occurring before maturity of the CDS. Using the previously outlined building blocks, we can formalize the value of a CDS in the following proposition.

**Proposition 2** *Assume a CDS involves receiving an amount  $P - \psi P$  if  $\mathcal{T} < T^*$ , and paying a continuous premium  $q$  until  $\min(T^*, \mathcal{T})$ . The value of the CDS is*

$$\text{CDS}(\omega_t, t) = (P - \psi P) \cdot G(\omega_t, t; T^*) - \frac{Q}{r} (1 - H(\omega_t, t; T^*) - G(\omega_t, t; T^*))$$

The first leg of the CDS-formula captures the value of receiving the bond's face value in case of default. The second leg captures the cost of paying the premium as a risk-free, infinite stream ( $\frac{Q}{r}$ ) less two terms: the first ( $H$ ) reflecting the discount attributable to the finite maturity of the swap, and the second ( $G$ ) reproducing the discount due to disrupted payments when and if default occurs.

Typically, the fee is chosen so that the credit default swap upon initiation ( $t = 0$ ) has zero value:

$$Q = \frac{r \cdot (P - \psi P) \cdot G(\omega_0, 0; T^*)}{(1 - H(\omega_0, 0; T^*) - G(\omega_0, 0; T^*))} \quad (2)$$

Often the fee is expressed as a fraction of the reference obligation's face value, and we will refer to the ratio  $q = \frac{Q}{P}$  as the credit default swap premium.

Intuitively, holding a CDS together with the reference obligation is close to holding the corresponding risk-free bond only. The positions are not identical, however, since the CDS typically has a different maturity and assures its holder the nominal amount ( $P$ ), rather than value of the risk free bond ( $B$ ), upon default. Yet, it is often convenient to think of the default swap premium of a just initiated swap as akin to the spread on the underlying corporate bond.

### 3 Empirical Method

In the previous section we laid down the pricing formulae for stocks, credits default swaps and bonds. In this section we discuss issues related to the practical implementation of our framework. Table 1 lists the notation used and the assigned parameter values; these are discussed below.

The following inputs are needed to price bonds and CDS using Propositions 1 and 2:

- the bond’s principal amount,  $P$ , the coupons  $c$ , maturity  $T$  and the coupon dates
- the recovery rate of the bond,  $\psi$
- the risk-free interest rate,  $r$
- the total nominal amount of debt,  $N$ , coupon  $C$  and maturity  $\Upsilon$  (Leland & Toft only)
- the bargaining power of debtholders  $\eta$  (Fan & Sundaresan only)
- the costs of financial distress,  $\alpha$
- the tax rate,  $\tau$
- the rate,  $\beta$ , at which earnings are generated by the assets, and finally
- the current value,  $\omega$ , and volatility of assets,  $\sigma$

Details of the bond contract are readily observable. However, the recovery rate of the bond in financial distress is not. We set it equal to 40%, roughly consistent with average defaulted debt recovery rate estimates for US entities between 1985-2001. For the risk-free rate we use constant maturity Treasury yields interpolated to match the maturity of the corporate bond or the CDS.

The nominal amount of debt equals the total liabilities taken from the firms’ balance sheets. For simplicity, we assume that the average coupon paid out to all the firm’s debt holders equates the risk-free rate:  $C = r \cdot N$ . To apply the LT model, we also need to specify the maturity of newly issued debt,  $\Upsilon$ . This choice turns out to be important and at the same time difficult to pin down; therefore, we will display results for three choices of maturity: 5, 6.76<sup>7</sup> and 10 years. In Fan & Sundaresan, maturity is, by design, infinite but in contrast, we need the bargaining power of debtholders – we use 0.5. Finally, we assume that 15% of the firm’s assets are lost in financial distress before being paid out to debtholders and fix the tax rate at 20%.<sup>8</sup>

The cash flow parameter  $\beta$  is of crucial importance. We, therefore, opt for a dual approach. The first assigns its value exogenously (using 0% and 6%), and the other predicts it as a weighted average of the historical dividend yield and relative interest expense. The average of weighted cash flow parameter  $\beta$  is 2.65% in our sample.

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<sup>7</sup>We consider the average debt maturity (3.38 years) reported by Stohs and Mauer (1994) which represents a reasonable average maturity of new debt ( $6.76 = 3.38 * 2$  years).

<sup>8</sup>The choice of 15% distress costs lies within the range estimated by Andrade and Kaplan (1998). The choice of 20% for the effective tax rate is consistent with the previous literature (see e.g. Leland (1998)) and is intentionally lower than the corporate tax rate to reflect personal tax benefits to equity returns, thus reducing the tax advantage of debt.

We then require estimates of asset value and volatility. The methodology utilized, first proposed by Duan (1994) in the context of deposit insurance, uses price data from one or several derivatives written on the assets to infer the characteristics of the underlying, unobserved, process. In principle, the "derivative" can be any of the firm's securities but in practice, only equity is likely to offer a precise and undisrupted price series.

The maximum likelihood estimation relies on a time series of stock prices,  $E^{obs} = \{\mathcal{E}_i^{obs} : i = 1 \dots n\}$ . A general formulation of the likelihood function using a change of variables is documented in Duan (1994). If we let  $w(\mathcal{E}_i^{obs}, t_i; \sigma) \equiv E^{-1}(\mathcal{E}_i^{obs}, t_i; \sigma)$  be the inverse of the equity function, the likelihood function for equity can be expressed as

$$L_{\mathcal{E}}(\mathcal{E}^{obs}; \sigma) = L_{\ln \omega}(\ln w(\mathcal{E}_i^{obs}, t_i; \sigma) : i = 2 \dots n; \sigma) - \sum_{i=2}^n \ln \omega_i \left. \frac{\partial \mathcal{E}(\omega_i, t_i; \sigma)}{\partial \omega_i} \right|_{\omega_i = w(\mathcal{E}_i^{obs}, t_i; \sigma)} \quad (3)$$

$L_{\ln \omega}$  is the standard likelihood function for a normally distributed variable, the log of the asset value, and  $\frac{\partial \mathcal{E}_i}{\partial \omega_i}$  is the "delta" of the equity formula.

An estimate of the asset values is computed using the inverse equity function:  $\hat{\omega}_t = w(\mathcal{E}_n^{obs}, t_n; \hat{\sigma})$ . Once we have obtained the pair  $(\hat{\omega}_t, \hat{\sigma})$  it is straightforward to compute the estimated CDS fee using (2). The bond spread is calculated by solving the bond price formula in Proposition 1, computing the risky yield, and subtracting the yield for the corresponding risk free bond.<sup>9</sup>

## 4 Data

To perform our estimation, we require price data on credit default swaps and corporate bonds as well as balance sheet and term structure information.

CreditTrade (CT) market prices are credit default swap (CDS) bids and offers that have either been placed directly into their electronic trading platform by traders, or entered into their database by their voice brokers who receive orders by telephone. Our database includes quotes and trades from June 1997 to April 2003. However, the early years' volumes are minimal and only as of 1999 does the volume become significant. The database distinguishes between bid and offers, and between quotes and trades. The evolution of credit ratings of the underlying debt by Moody's and S&P is recorded from COMPUSTAT. In the early part of our sample, restructuring is considered

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<sup>9</sup>This bond is valued as a hypothetical bond with the same promised payments as the risky bonds but with yield equal to a linear interpolation of bracketing constant maturity Treasury (CMT) yield indices.

a credit event. From the very end of 2002 onwards, restructuring is no longer considered a credit event of the CDSs. The underlying debt is almost exclusively senior. All contracts are USD denominated. For consistency, we retain only CDS on senior unsecured debt with restructuring as default event. Very little information is lost with the use of each of these filters.

Our bond transaction data is sourced from the National Association of Insurance Commissioners (NAIC). Bond issue - and issuer - related descriptive data are obtained from the Fixed Investment Securities Database (FISD). The majority of transactions in the NAIC database take place between 1994 and 2003.

Cleaning-up of the raw NAIC database was carried out in three steps. In the first, bond transactions with counterparty names other than recognized financial institutions are excluded. Transactions without a clearly defined counterparty are deemed unreliable.

In the second step, we restricted our sample to fixed coupon rate USD denominated bonds with issuers in the industrial sector. Furthermore, we eliminated bond issues with option features, such as callables, puttables, and convertibles. Asset-backed issues, bonds with sinking funds or credit enhancements were also removed to ensure bond prices in the sample truly reflect the underlying credit quality of issuers.

The third step involves selecting those bonds for which we have their issuers' complete and reliable market capitalizations as well as accounting information about liabilities. Daily equity values are obtained from DATASTREAM. Quarterly firm balance sheet data is taken from COMPUSTAT.

In total, 145 firms qualify in both the CT and NAIC databases. When we take an intersection of the CT and NAIC data, while requiring a transaction and / or quote for the bonds and default swaps on the same name during the same day, we are left with 1387 pairs from 116 distinct entities.<sup>10</sup>

Table 2 identifies descriptive statistics for firm, bond and CDS features. In 2a we see that firm sizes vary from 2.0 billion to 78.4 billion with an average of 67.9 billion dollars. Bond issuers' / CDS reference entities' S&P credit ratings range between AA and CCC while the majority lies between BBB+ and BBB. The average bond issue size is 593 million dollars, with 0.65 million and 5,000 million as extremes. The average transaction size is approximately 5.9 million dollars. On average, bonds were 3.9 years of age and had 9.7 years remaining to maturity. The average coupon rate is 6.9 percent across all issues. The longest maturity of CDS in our sample is 10 years

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<sup>10</sup>Note that since we are not using the bond prices and CDS premia in our estimation, we do not require any matching or bracketing between bond and CDS maturities.

versus 0.3 years as the shortest. The average maturity is 4.9 years.

Table 2b presents the distribution of market CDS and bond spreads over different ratings. As expected, the CDS premia, like bond yield spreads, are largely determined by the reference entity's / issuer's credit quality. Moreover, when Treasury rates are used as riskfree benchmark, bond spreads lead over CDS premia across all rating categories save one (BB-, where there are only five observations). CDS premia only represent 33% of bond spreads for the AA range, but this fraction increases steadily to ca. 81% for a BB rating. For lower ratings, for which we have less data, the pattern is no longer as clear, but the ratio remains much higher than for high grade issuers. This is suggestive of a proportionally larger non-default component in bond yield spreads over Treasury rates for issuers with little default risk. This pattern largely disappears when swap rates are used

## 5 Results

We first briefly characterize the outcome of implementing each model in terms of among other measures the asset value and volatility estimates, before turning to the main results – the corresponding predicted CDS premia and bond spreads. Table 4 reports these estimates when the Treasury curve is used as benchmark (Panel A) and when the swpa curve is used (Panel B). Both panels show that the Leland (1994) (L) and Fan and Sundaresan (2000) (FS) models yield approximately the same asset value and volatility estimates on average, whereas the Leland and Toft (1996) (LT) model produces higher asset value but lower asset volatility. The reason is that the higher barrier in the latter model, *ceteris paribus*, increases the theoretical equity volatility and lowers equity value. Hence the model will predict a higher asset value and/or a lower volatility to match observed equity volatilities. As a measure of the combined effect of asset value and volatility we also report a KMV-style distance-to-default, i.e.  $\frac{\omega - L}{\sigma\omega}$ . This shows that the higher asset value and lower volatility estimates in the LT model are sufficient to compensate the effect of the higher barrier; the model produces the same distance-to-default as the Leland model does (2.8), which the FS model has the highest (2.7). The LT model reports the highest the average values of dollar-in-defaults of 2 and 10 years maturity. Therefore, the LT model is expected to produce the highest CDS / bond spreads, followed by FS and finally L.

It is interesting to note that the choice of benchmark curve has a negligible effect on the estimation of asset value and volatility. Nevertheless, as we will see shortly, the choice is critical to the measurement of bond yield spreads and thus to assess model performance.

Table 5 reports the estimated asset value volatilities of various credit rating by the LT model. Our results of 23% on average closely resemble that of 22% in Schaefer and Strebulaev (2004). Although there are some differences across rating categories, for those where the bulk of our data lies (A and BBB ratings) we are well aligned.

In addition Figure 1 plots the time series of average asset values, volatilities, leverage and market capitalization across the firms in the sample. At least two patterns become apparent. First, volatility was higher between 1999 and 2001. This is visible not only in Panel A that plots asset volatility but also in Panels B and C which summarize asset and equity values. At the same time, leverage increases during the beginning of our sample and appears to stabilize at around 50% as of the second half of 2001.

Notice that the time series of these means appear highly volatile. This can at least partly be attributed to the time varying composition of our transaction data: two consecutive means are unlikely to derive from the same (small) subset of firms.

Now we turn our attention to CDS premia estimated by the three models, reported in tables 6-8, together with results for bond spreads. Using Treasury rates as riskfree benchmark, the L, FS and LT models produce mean CDS premia of 69, 93 and 110 bps respectively compared to the observed mean CDS premium of 112 bps. Thus, our perhaps most plausible model specification produces CDS premia close to market quotes. The results show that the estimated CDS premia are not significantly affected by the choice of riskfree rates. The L, FS and LT models produce mean CDS premia of 62, 88 and 107 bps respectively, using swap rates as riskfree input. In addition, it is interesting to note that the mean market CDS premium 112 bps is almost identical to the mean market bond spreads 111 bps when swap rates are used as riskfree rates.

As expected, when Treasury rates are used as riskfree benchmark, the L model estimates the lowest mean bond spread, 82 bps, while the FS and LT models estimate mean bond spreads of 107 bps and 114 bps, respectively. Compared to the mean market bond spread of 173 bps, they substantially underestimate bond yield spreads by 53%, 38% and 34%. This is in line with the findings of previous studies<sup>11</sup>. However, when swap rates are used as riskfree benchmark, the L, FS and LT models estimate mean bond spreads of 73, 99 and 109 bps respectively, in comparison to the mean market bond spread of 111 bps. They underestimate bond yield spreads by 34%, 11% and 2%. The LT model produces fairly accurate spread predictions on average in this case.

The pricing of CDS contracts is tied to that of corporate bonds by an arbitrage argument. A

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<sup>11</sup>See for example Jones, Mason, and Rosenfeld (1984), Ogden (1987) and Lyden and Saranati (2000). The numbers are also comparable to what Huang and Huang (2002) find for various models in an extensive calibration exercise.

package of a corporate par floating rate note and default protection through a CDS is theoretically a risk free investment. Given that a CDS contract is unfunded, i.e. requires no up front payment, comparing bond yield spreads to CDS spreads requires taking into account the funding rate of a bondholder, which is likely in the vicinity of swap rates. Thus, in practice, this implies that the bond spread most comparable to the CDS price is the spread to the swap curve. Given that structural models do not explicitly account for funding above the commonly used proxies for the risk free rate, it would seem reasonable to expect them to perform better on CDS than on bonds, where funding does not need to be adjusted for. Using the swap curve can then be viewed as a pragmatic approach to correcting for the cost of funding when implementing a structural model for bonds.

To assess the sensitivity of our results we also report estimated premia for alternative values of the cash flow rate and, for the LT model, bond maturity. The cash flow rate tends to increase bond spreads and CDS premia because firms with higher cash payout rates grow less and thus have higher default probabilities. For instance, when Treasury are used as riskfree rates, increasing  $\beta$  can help to reduce the yield spreads and CDS premia underestimation for the Leland and FS models, although not eliminate it. For the LT model, bond yield spread underestimation persists. For CDS premia, the parameter can tilt the balance although at 6%, CDS premia are clearly overstated. The weighted average method of estimating  $\beta$ , provides reasonable results for CDS premia in the LT model.

Note that both 0% and 6% are extreme values. Few if any firms will issue debt and finance coupons entirely out of newly raised equity. A payout rate of 6% is more than two times higher than the typical weighted average between firms' dividend yields and interest expenses. Thus numbers reported at these values should illustrate the highest and lowest spreads / premia that can be obtained by varying this parameter.

For debt maturity in the LT model we consider three values: 5, 6.76 and 10 years. Note that the maturity parameter in this model represents the maturity of newly issued debt. It should, therefore, exceed the average maturity of a firm's debt. On the other hand, a firm's debt consists not only of bonds but also of a variety of credits with very short maturity. Consequently, although many firms issue bonds with maturities exceeding 10 years, they are likely to also issue medium-term notes, short-term commercial paper, while also indirectly borrowing from their suppliers, the government and their employees. It seems unlikely that the average maturity of new debt exceeds ten years.

Figure 2 plots the time series of average model and market bond and CDS spreads for the two

benchmark curves using the benchmark LT specification. It seems that on average the models do a reasonable job of matching market spreads for CDS and for bonds when the swap curve is used as benchmark. The exception is during 2000, when the model overshoots. Interestingly this precedes a period with a historically very high incidence of defaults.

Next, we consider the residuals, defined as the difference between market and model bond or CDS spreads. Figure 3 plots the time series of the difference between the average bond and CDS residuals for the two choices of benchmark yield curve. The difference between the residuals ( $R$ ) for the Treasury curve would equal  $R_{bond} - R_{CDS} = y_{Mkt} - y_T - (y_{Mod} - y_T) - (CDS_{Mkt} - CDS_{Mod})$  where  $y_{Mkt}$  and  $y_{Mod}$  denote the market and model bond yields respectively,  $y_T$  the corresponding Treasury yield and  $CDS_{Mkt/Mod}$  the default swap spreads. The model predicted bond and CDS spreads are approximately the same for a given maturity.<sup>12</sup> Hence  $R_{bond} - R_{CDS} = y_{Mkt} - y_T - CDS_{Mkt}$ . This can be rewritten  $R_{bond} - R_{CDS} = (y_{Mkt} - y_S) - CDS_{Mkt} + (y_S - y_T)$ , where  $y_S$  denotes the swap yield. The last component,  $(y_S - y_T)$ , corresponds to the swap spread and whereas the (negative of the) first is known as the CDS basis by practitioners. This is a measure of relative pricing in the cash and derivative markets for default risk. It would be interesting in itself to study this variable but this lies outside the scope of this paper. What is important for our purposes is the swap spread component. It suggests that the difference in residuals relative to the Treasury curve will largely be driven by this spread.<sup>13</sup> The top panel in Figure 3 confirms this by showing that when the swap spread is high, the difference in the residuals is high and vice versa. The bottom panel plots the difference in residuals when the swap curve is the chosen benchmark. In that case, the above argument suggests that the main driver should be the basis.

To better understand the drivers of the residuals, we perform a linear regression analysis of their cross section and time series. The results of which are reported in Tables 9 & 10. Table 9 summarizes the impact of common variables on the time series of average residuals for bonds and CDS using both benchmark curves. The explanatory variables are the 5 year CMT rate, the default premium as measured by the difference between Moody's Baa and Aaa spread indices, the 5 year swap rate, the S&P 500 return and the VIX. Only one variable is systematically statistically significant - the S&P 500 return. This result may seem surprising given that individual firms' equity prices are inputs to the estimation of bond and CDS premia. One interpretation is that the information in equity prices for contemporaneous bond and CDS spreads may not be sufficiently accurate about the default risk premium component of the spread and that this is captured by

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<sup>12</sup>The coupon rate also influences this relationship as the arbitrage is only exact for a par floater.

<sup>13</sup>We thank Stephen Schaefer for this insight.

the market return variable. For bonds residuals, the default premium is significant but this is not the case for CDS residuals. This could be a sign of the presence of a bond market specific risk which does not spill over to the CDS market.

To attempt to explain the cross section of residuals, we average across time for each firm. Then firm specific averages are regressed on bond and firm specific variables: the bond coupon rate, the bond transaction size, the size of the bond issue, firm size, bond maturity and a dummy that measures bond age (OTR). None of these variables are significant for residuals of either bonds or CDS. The explanatory power is consistently low for these regressions.

## 6 Concluding remarks

Using a set of structural models, we have evaluated the price of default protection for a sample of US corporations. We find that one of our studied models has little difficulty in predicting default swap premia on average. This result departs from what has been found in corporate bond markets. For robustness, we perform the same exercise for bond spreads by the same issuers on the same trading date. As previous work has found, bond spreads relative to the Treasury curve are systematically underestimated. However, this is not the case when the swap curve is used as a benchmark, suggesting that previously documented underestimation results may be sensitive to the choice of risk free rate. A reason why the swap curve may be a more appropriate benchmark for corporate bond spread measurement is that it lies closer to the cost of funding for traders in the bond market. The bond spread over the swap curve thus measures the additional yield these market participants require to participate in this market when faced with the alternative of dealing in credit derivatives.

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# A Appendix

## A.1 Building blocks for CDS and bonds

First, define default as the time ( $\mathcal{T}$ ) the asset value hits the default boundary from above,  $\ln \frac{\omega_{\mathcal{T}}}{L_{\mathcal{T}}} \equiv 0$ . Then define  $G(\omega_t, t)$  as the value of a claim paying off \$1 in default:

$$G(\omega_t, t) \equiv E^B [e^{-r(\mathcal{T}-t)} \cdot 1]$$

We let  $E^B$  denote expectations under the standard pricing measure. The value of  $G$  is given by

$$G(\omega_t, t) = \left( \frac{\omega_t}{L_t} \right)^{-\theta}$$

with the constant given by

$$\theta = \frac{\sqrt{(h^B)^2 + 2r + h^B}}{\sigma}$$

and

$$h^B = \frac{r - \beta - 0.5\sigma^2}{\sigma}$$

Define the dollar-in-default with maturity  $G(\omega_t, t; T)$  as the value of a claim paying off \$1 in default *if* it occurs before  $T$

$$G(\omega_t, t; T) \equiv E^B [e^{-r(\mathcal{T}-t)} \cdot 1 \cdot (1 - I_{\mathcal{T} \leq T})]$$

and define the binary option  $H(\omega_t, t; T)$  as the value of a claim paying off \$1 at  $T$  if default has *not* occurred before that date

$$H(\omega_t, t; T) \equiv E^B [e^{-r(T-t)} \cdot 1 \cdot I_{\mathcal{T} > T}]$$

$I_{\mathcal{T} > T}$  is the indicator function for the *survival event*, i.e. the event that the asset value ( $\omega_T$ ) has not hit the barrier prior to maturity ( $\mathcal{T} \not\leq T$ ). The price formulae for the last two building blocks are given below. They contain a term that expresses the probabilities (under different measures) of the survival event – or, the *survival probability*. To clarify this common structure, we first state those probabilities in the following lemma.<sup>14</sup>

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<sup>14</sup>The probabilities are previously known, as is the formula the down-and-out binary option in Lemma A.1 (see for example Björk (1998)).

**Lemma 1** *The probabilities of the event  $(\mathcal{T} \not\leq T)$  (the “survival event”) at  $t$  under the probability measures  $Q^m : m = \{B, G\}$  are*

$$Q^m(\mathcal{T} \not\leq T) = \phi\left(k^m\left(\frac{\omega_t}{L_t}\right)\right) - \left(\frac{\omega_t}{L_t}\right)^{-\frac{2}{\sigma}h^m} \phi\left(k^m\left(\frac{L_t}{\omega_t}\right)\right)$$

where

$$\begin{aligned} k^m(x) &= \frac{\ln x}{\sigma\sqrt{T-t}} + h^m\sqrt{T-t} \\ h^G &= h^B - \theta \cdot \sigma = -\sqrt{(h^B)^2 + 2r} \end{aligned}$$

$\phi(k)$  denotes the cumulative standard normal distribution function with integration limit  $k$ .

The probability measure  $Q^G$  is the measure having  $G(\omega_t, t)$  as numeraire (the Girsanov kernel for going to this measure from the standard pricing measure is  $\theta \cdot \sigma$ ). Using this lemma we obtain the pricing formulae for the building blocks in a convenient form. The price of a down-and-out binary option is

$$H(\omega_t, t; T) = e^{-r(T-t)} \cdot Q^B(\mathcal{T} \not\leq T)$$

The price of a dollar-in-default claim with maturity  $T$  is

$$G(\omega_t, t; T) = G(\omega_t, t) \cdot (1 - Q^G(\mathcal{T} \not\leq T))$$

To understand this second formula, note that the value of receiving a dollar if default occurs prior to  $T$  must be equal to receiving a dollar-in-default claim with infinite maturity, less a claim where you receive a dollar in default conditional on it *not* occurring prior to  $T$ :

$$G(\omega_t, t; T) = G(\omega_t, t) - e^{-r(T-t)} E^B[G(\omega_T, T) \cdot I_{\mathcal{T} \not\leq T}]$$

Using a change of probability measure, we can separate the variables within the expectation brackets (see e.g. Geman, El-Karoui, and Rochet (1995)).

$$\begin{aligned} G(\omega_t, t; T) &= G(\omega_t, t) - e^{-r(T-t)} E^B[G(\omega_T, T)] \cdot E^G[I_{\mathcal{T} \not\leq T}] \\ &= G(\omega_t, t) \cdot (1 - Q^G(\mathcal{T} \not\leq T)) \end{aligned}$$

## A.2 The Leland Model

The value of the firm

$$\mathcal{F}(\omega_t) = \omega_t + \tau \frac{C}{r} \left[ 1 - \left( \frac{\omega_t}{L} \right)^{-x} \right] - \alpha L \cdot \left( \frac{\omega_t}{L} \right)^{-x}$$

with

$$x = \frac{(r - \beta - 0.5\sigma^2) + \sqrt{(r - \beta - 0.5\sigma^2)^2 + 2\sigma^2 r}}{\sigma^2}$$

Value of debt

$$\mathcal{D}(\omega_t) = \frac{C}{r} + \left( (1 - \alpha) L - \frac{C}{r} \right) \left( \frac{\omega_t}{L} \right)^{-x}$$

The bankruptcy barrier

$$L = \frac{(1 - \tau)C}{r} \frac{x}{1 + x}$$

## A.3 The Leland & Toft Model

The value fo the firm is the same as in Leland (1994). The value of debt is given by

$$\mathcal{D}(\omega_t) = \frac{C}{r} + \left( N - \frac{C}{r} \right) \left( \frac{1 - e^{-r\Upsilon}}{r\Upsilon} - I(\omega_t) \right) + \left( (1 - \alpha) L - \frac{C}{r} \right) J(\omega_t)$$

The bankruptcy barrier

$$L = \frac{\frac{C}{r} \left( \frac{A}{r\Upsilon} - B \right) - \frac{AP}{r\Upsilon} - \frac{\tau Cx}{r}}{1 + \alpha x - (1 - \alpha)B}$$

where

$$\begin{aligned} A &= 2ye^{-r\Upsilon} \phi \left[ y\sigma\sqrt{\Upsilon} \right] - 2z\phi \left[ z\sigma\sqrt{\Upsilon} \right] \\ &\quad - \frac{2}{\sigma\sqrt{\Upsilon}} n \left[ z\sigma\sqrt{\Upsilon} \right] + \frac{2e^{-r\Upsilon}}{\sigma\sqrt{\Upsilon}} n \left[ y\sigma\sqrt{\Upsilon} \right] + (z - y) \\ B &= - \left( 2z + \frac{2}{z\sigma^2\Upsilon} \right) \phi \left[ z\sigma\sqrt{\Upsilon} \right] - \frac{2}{\sigma\sqrt{\Upsilon}} n \left[ z\sigma\sqrt{\Upsilon} \right] + (z - y) + \frac{1}{z\sigma^2\Upsilon} \end{aligned}$$

and  $n[\cdot]$  denotes the standard normal density function.

The components of the debt formulae are

$$I(\omega) = \frac{1}{r\Upsilon} \left( i(\omega) - e^{-r\Upsilon} j(\omega) \right)$$

$$i(\omega) = \phi[h_1] + \left( \frac{\omega}{L} \right)^{-2a} \phi[h_2]$$

$$j(\omega) = \left(\frac{\omega}{L}\right)^{-y+z} \phi[q_1] + \left(\frac{\omega}{L}\right)^{-y-z} \phi[q_2]$$

and

$$J(\omega) = \frac{1}{z\sigma\sqrt{\Upsilon}} \begin{pmatrix} -\left(\frac{\omega}{L}\right)^{-a+z} \phi[q_1] q_1 \\ + \left(\frac{\omega}{L}\right)^{-a-z} \phi[q_2] q_2 \end{pmatrix}$$

Finally

$$q_1 = \frac{-b - z\sigma^2\Upsilon}{\sigma\sqrt{\Upsilon}}$$

$$q_2 = \frac{-b + z\sigma^2\Upsilon}{\sigma\sqrt{\Upsilon}}$$

$$h_1 = \frac{-b - y\sigma^2\Upsilon}{\sigma\sqrt{\Upsilon}}$$

$$h_2 = \frac{-b + y\sigma^2\Upsilon}{\sigma\sqrt{\Upsilon}}$$

and

$$y = \frac{r - \beta - 0.5\sigma^2}{\sigma^2}$$

$$z = \frac{\sqrt{y^2\sigma^4 + 2r\sigma^2}}{\sigma^2}$$

$$x = y + z$$

$$b = \ln\left(\frac{\omega}{L}\right)$$

#### A.4 The Fan & Sundaresan Model

Given the trigger point for strategic debt service  $S$ , the value of the firm is

$$\mathcal{F}(\omega_t) = \begin{cases} \omega_t + \frac{\tau C}{r} - \frac{\lambda_+}{\lambda_+ - \lambda_-} \frac{\tau C}{r} \left(\frac{\omega_t}{S}\right)^{\lambda_-}, & \text{when } \omega_t > S \\ \omega_t + \frac{-\lambda_-}{\lambda_+ - \lambda_-} \frac{\tau C}{r} \left(\frac{\omega_t}{S}\right)^{\lambda_+}, & \text{when } \omega_t \leq S \end{cases}$$

The equity value is

$$\mathcal{E}(\omega_t) = \begin{cases} \omega_t - \frac{C(1-\tau)}{r} + \left[ \frac{C(1-\tau)}{(1-\lambda_-)r} - \frac{\lambda_-(1-\lambda_+)\eta}{(\lambda_+-\lambda_-)(1-\lambda_-)} \frac{\tau C}{r} \right] \left( \frac{\omega_t}{S} \right)^{\lambda_-}, & \text{when } \omega_t > S \\ \eta \mathcal{F}(\omega_t) - \eta(1-\alpha)\omega_t, & \text{when } \omega_t \leq S \end{cases} \quad (4)$$

where  $\eta$  denotes bargaining power of bondholders. The trigger point for strategic debt service is

$$S = \frac{(1-\tau+\eta\tau)C}{r} \frac{-\lambda_-}{1-\lambda_-} \frac{1}{1-\eta\alpha}$$

and

$$\lambda_{\pm} = 0.5 - \frac{(r-\beta)}{\sigma^2} \pm \sqrt{\left[ \frac{(r-\beta)}{\sigma^2} - 0.5 \right]^2 + \frac{2r}{\sigma^2}}$$

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TABLE 1: LIST OF NOTATION / PARAMETER VALUES

Parameter	Notation	Assigned Value		
		Leland	Leland & Toft	Fan & Sundaresan
Bond principal / coupon / maturity	$P / c / T$	According to actual contract		
CDS and bond recovery rate	$\psi$	40%	40%	40%
Riskfree rate	$r$	Treasury yields interpolated to match bond maturity		
Debt Nominal	$N$	Total liabilities from firm's balance sheet		
Debt Coupon	$C$	$r \cdot N$	$r \cdot N$	$r \cdot N$
Debt Maturity	$T$	—	5 / 6.76 / 10	—
Bargaining power of debtholders	$\eta$	—	—	0.5
Costs of Financial Distress	$\alpha$	15%	15%	15%
Tax rate	$\tau$	20%	20%	20%
Revenue rate	$\beta$	0% / Weighted average <sup>1</sup> / 6%		

<sup>1</sup>The weighted average of the revenue rate is calculated from the balance sheets as

$$\left( \frac{\text{Interest Expenses}}{\text{Total Liabilities}} \right) \times \text{Leverage} + (\text{Dividend Yield}) \times (1 - \text{Leverage})$$

where

$$\text{Leverage} = \frac{\text{Total Liabilities}}{(\text{Total Liabilities} + \text{Market Capital})}$$

TABLE 2: DESCRIPTIVE STATISTICS - FIRMS, BONDS AND DEFAULT SWAPS

Variable	Mean	Std. Dev.	Minimum	Maximum
Firm Size (billions of \$)	67.9	96.2	2.0	78.4
Leverage	50%	19%	8%	95%
SP rating	9.8	4.8	1	27
Bond transaction size (dollars)	5.9"	20.3"	3000	411"
Bond issue size (dollars)	593"	660"	652'	5000"
Bond age	3.9	3.4	0	15.9
Bond maturity	9.7	8.6	1.1	98.0
Bond coupon	6.9	1.2	3.4	11
CDS maturity	4.9	0.8	0.3	10.0

TABLE 3: SUMMARY STATISTICS - BOND SPREADS AND CDS PREMIA

Rating	Transactions	Bond Yield spreads relative Treasury curve				Bond Yield spreads relative swap curve				CDS premia			
		Mean	Stdev	Min	Max	Mean	Stdev	Max	Min	Mean	Stdev	Max	Min
AAA	19	90	18	29	106	40	31	-33	132	58	21	10	71
AA	43	75	28	8.5	150	4	29	-46	93	25	12	7	59
AA-	30	61	23	25	117	-1	13	-21	38	24	8	12	47
A+	95	96	35	13	172	33	42	-42	168	44	21	15	105
A	170	118	43	26	281	57	47	-57	223	51	26	15	140
A-	222	152	47	64	394	97	57	-10	396	92	48	27	410
BBB+	331	146	60	34	552	85	62	-24	480	94	50	11	375
BBB	201	189	88	44	657	125	94	-34	598	114	81	22	503
BBB-	93	302	163	97	914	248	172	53	894	225	154	35	825
BB+	36	367	131	230	808	298	144	136	760	299	176	110	750
BB	2	356	146	252	460	278	134	183	373	338	37	313	365
BB-	5	446	290	84	788	380	298	37	737	463	298	19	729
B+	20	254	192	53	710	180	204	-6	683	220	258	22	825
B	17	327	166	134	857	243	175	70	801	205	193	55	777
B-	36	194	125	43	546	106	137	-23	463	129	190	15	900
CCC	1	369	na	na	na	282	na	na	na	177	na	na	na

TABLE 4: SUMMARY OF FIRM SPECIFIC ESTIMATION OUTPUT

Panel A: Treasury reference curve

Variable	Model	Mean	Std.Dev.	Min	Max
Asset return volatility	L	30%	12%	6%	90%
	FS	30%	12%	6%	91%
	LT (T=6.76)	23%	11%	6%	73%
Asset value (billion of \$)	L	58.0	81.4	1.0	617.4
	FS	57.8	80.7	1.0	607.8
	LT (T=6.76)	64.3	90.4	1.9	720.5
Default barrier (billion of \$)	L	10.7	22.4	0.1	215.4
	FS	11.4	23.6	0.1	226.5
	LT (T=6.76)	20.8	43.6	0.6	420.6
Distance to default	L	2.8	0.8	1.0	5.4
	FS	2.7	0.7	1.0	5.0
	LT (T=6.76)	2.8	0.7	1.1	5.5
Dollar in default year 2 (%)	L	0.8%	2.7%	0.0%	28.8%
	FS	1.3%	3.8%	0.0%	37.9%
	LT (T=6.76)	1.4%	3.3%	0.0%	37.4%
Dollar in default year 10 (%)	L	12.8%	13.4%	0.0%	79.3%
	FS	15.1%	14.6%	0.0%	80.6%
	LT (T=6.76)	17.2%	14.0%	0.0%	83.8%

Panel B: Swap reference curve

Variable	Model	Mean	Std.Dev.	Min	Max
Asset return volatility	L	29%	12%	7%	85%
	FS	29%	12%	7%	86%
	LT (T=6.76)	23%	11%	6%	73%
Asset value (billion of \$)	L	59.1	83.4	1.1	643.1
	FS	58.8	82.8	1.1	635.4
	LT (T=6.76)	64.0	89.8	1.9	714.8
Default barrier (billion of \$)	L	12.0	26.1	0.2	251.4
	FS	12.8	27.6	0.2	265.6
	LT (T=6.76)	20.6	42.7	0.5	411.8
Distance to default	L	2.8	0.8	1.0	5.3
	FS	2.7	0.7	1.0	5.0
	LT (T=6.76)	2.8	0.7	1.1	5.2
Dollar in default year 2 (%)	L	0.7%	2.7%	0.0%	29.7%
	FS	1.3%	3.9%	0.0%	39.8%
	LT (T=6.76)	1.4%	3.3%	0.0%	37.2%
Dollar in default year 10 (%)	L	11.2%	12.2%	0.0%	76.5%
	FS	13.6%	13.5%	0.0%	78.2%
	LT (T=6.76)	15.9%	13.2%	0.0%	82.8%

TABLE 5: ASSET VOLATILITY ESTIMATES BY RATING

	CMT	SWAP	Schaefer & Strebulaev
AAA	16%	17%	25%
AA	29%	29%	22%
A	21%	22%	22%
BBB	23%	23%	22%
BB	25%	25%	26%
B	36%	36%	31%
CCC	26%	26%	38%
all	23%	23%	22%

TABLE 6: BOND SPREAD AND CDS PREMIA ESTIMATES: THE LELAND MODEL

Panel A: Treasury reference curve

	Market Bond Spreads	Model Bond Spreads	Residual Bond Spreads	Market CDS Premia	Model CDS Premia	Residual CDS Premia
beta = 0						
Mean	173	47	127	112	37	75
Std. Dev.	120	73	107	119	68	102
Min	9	0	-292	7	0	-345
Max	938	614	914	1000	588	905
beta = w. ave.						
Mean	173	82	91	112	69	43
Std. Dev.	120	123	115	119	134	124
Min	9	0	-506	7	0	-682
Max	938	983	914	1000	1070	699
beta = 0.06						
Mean	173	104	70	112	77	35
Std. Dev.	120	130	121	119	139	127
Min	9	0	-504	7	0	-624
Max	938	1223	914	1000	1209	728

Panel B: Swap reference curve

	Market Bond Spreads	Model Bond Spreads	Residual Bond Spreads	Market CDS Premia	Model CDS Premia	Residual CDS Premia
beta = 0						
Mean	111	44	67	112	36	76
Std. Dev.	124	70	113	119	66	102
Min	-57	0	-358	7	0	-346
Max	903	606	894	1000	575	916
beta = w. ave.						
Mean	111	73	38	112	62	50
Std. Dev.	124	117	118	119	126	118
Min	-57	0	-550	7	0	-655
Max	903	976	894	1000	1083	726
beta = 0.06						
Mean	111	96	15	112	73	40
Std. Dev.	124	123	121	119	131	121
Min	-57	0	-487	7	0	-562
Max	903	1166	894	1000	1152	744

TABLE 7: BOND SPREAD AND CDS PREMIA ESTIMATES: THE FAN &amp; SUNDARESAN MODEL

Panel A: Treasury reference curve						
	Market Bond Spreads	Model Bond Spreads	Residual Bond Spreads	Market CDS Premia	Model CDS Premia	Residual CDS Premia
beta = 0						
Mean	173	74	99	112	64	49
Std. Dev.	120	118	113	119	108	111
Min	9	0	-354	7	0	-464
Max	938	868	914	1000	642	801
beta = w. ave.						
Mean	173	107	67	112	93	19
Std. Dev.	120	158	136	119	167	147
Min	9	0	-808	7	0	894
Max	938	1293	914	1000	1346	672
beta = 0.06						
Mean	173	127	46	112	99	13
Std. Dev.	120	164	142	119	172	151
Min	9	0	-949	7	0	-964
Max	938	1668	914	1000	1549	674

Panel B: Swap reference curve

	Market Bond Spreads	Model Bond Spreads	Residual Bond Spreads	Market CDS Premia	Model CDS Premia	Residual CDS Premia
beta = 0						
Mean	111	72	39	112	63	49
Std. Dev.	124	122	121	119	108	109
Min	-57	0	-468	7	0	-455
Max	903	1003	894	1000	660	819
beta = w. ave.						
Mean	111	99	12	112	88	25
Std. Dev.	124	155	138	119	161	141
Min	-57	0	886	7	0	-892
Max	903	1453	894	1000	1401	672
beta = 0.06						
Mean	111	120	-9	112	96	16
Std. Dev.	124	160	143	119	167	146
Min	-57	0	-989	7	0	-945
Max	903	1668	894	1000	1529	671

TABLE 8: BOND SPREAD AND CDS PREMIA ESTIMATES: THE LELAND &amp; TOFT MODEL

## Panel A: Treasury reference curve

	Market Bond Spreads	Model Bond Spreads	Residual Bond Spreads	Market CDS Premia	Model CDS Premia	Residual CDS Premia	Market Bond Spreads	Model Bond Spreads	Residual Bond Spreads	Market CDS Premia	Model CDS Premia	Residual CDS Premia
beta = 0, T=6.76						T = 5, beta =w.ave.						
Mean	173	84	89	112	80	33	173	126	47	112	125	-12
Std. Dev.	120	105	110	119	101	108	120	145	132	119	165	146
Min	9	0	-402	7	0	-543	9	0	-533	7	0	-961
Max	938	830	910	1000	869	860	938	1079	908	1000	1319	754
beta = w. ave., T=6.76						T = 6.76, beta =w.ave.						
Mean	173	114	59	112	110	2	173	114	59	112	110	2
Std. Dev.	120	133	124	119	150	135	120	133	124	119	150	135
Min	9	0	-510	7	0	-825	9	0	-510	7	0	-825
Max	938	895	910	1000	1183	782	938	895	910	1000	1183	782
beta = 0.06, T=6.76						T = 10, beta =w.ave.						
Mean	173	167	6	112	150	-38	173	101	72	112	94	18
Std. Dev.	120	157	142	119	171	144	120	123	118	119	136	124
Min	9	0	-604	7	0	-689	9	0	-502	7	0	-709
Max	938	1046	911	1000	1211	652	938	801	912	1000	1067	806

Panel B: Swap reference curve

	Market Bond Spreads	Model Bond Spreads	Residual Bond Spreads	Market CDS Premia	Model CDS Premia	Residual CDS Premia	Market Bond Spreads	Model Bond Spreads	Residual Bond Spreads	Market CDS Premia	Model CDS Premia	Residual CDS Premia
beta = 0, T=6.76						T = 5, beta =w.ave.						
Mean	111	78	33	112	76	37	111	120	-9	112	121	-8
Std. Dev.	124	100	115	119	97	106	124	141	134	119	160	141
Min	-57	0	-457	7	0	-529	-57	0	-591	7	0	-943
Max	903	780	889	1000	859	879	903	1171	888	1000	1302	769
beta = w. ave., T=6.76						T = 6.76, beta =w.ave.						
Mean	111	109	2	112	107	6	111	109	2	112	107	6
Std. Dev.	124	129	127	119	145	130	124	129	127	119	145	130
Min	-57	0	-560	7	0	-809	-57	0	-560	7	0	-809
Max	903	961	890	1000	1168	794	903	961	890	1000	1168	794
beta = 0.06, T=6.76						T = 10, beta =w.ave.						
Mean	111	160	-49	112	148	-36	111	96	15	112	92	20
Std. Dev.	124	155	145	119	170	144	124	118	121	119	131	120
Min	-57	0	-657	7	0	-690	-57	0	-552	7	0	-696
Max	903	1103	891	1000	1202	684	903	759	892	1000	1055	815

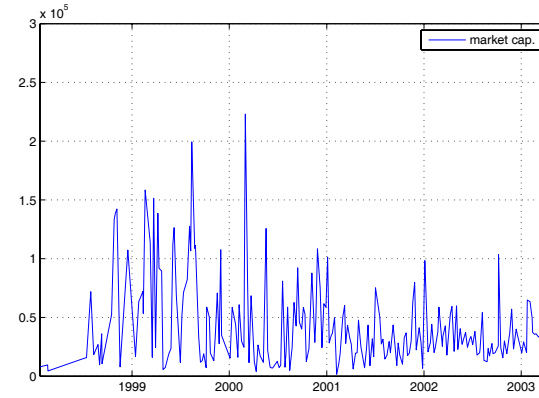
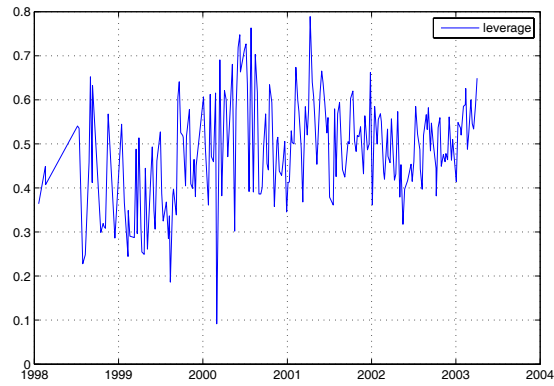
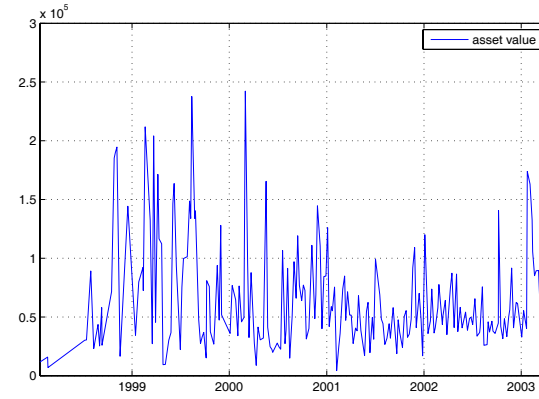
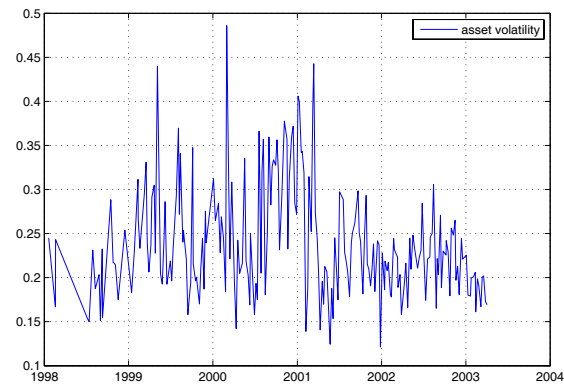
TABLE 9: RESIDUAL ANALYSIS (WEEKLY AVERAGE TIME SERIES)

(A) Bond Residual Spread (CMT)							(C) CDS Residual Premium (CMT)						
	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]			Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
CMT 5Y	-7.466	10.004	-0.75	0.456	-27.18	12.25		-15.54	10.034	-1.55	0.123	-35.32	4.23
Default Prem.	<b>0.6447</b>	<b>2.91E-01</b>	<b>2.22</b>	<b>0.028</b>	0.071	1.22		0.154	0.319	0.48	0.63	-0.475	0.783
Swap 5Y	-0.1047	0.339	-0.31	0.758	-0.774	0.5648		-0.582	0.379	-1.53	0.126	-1.331	0.165
SP500 Return	<b>311.9</b>	<b>99.243</b>	<b>3.14</b>	<b>0.002</b>	116.3	507.5		<b>317.3</b>	<b>93.705</b>	<b>3.39</b>	<b>0.001</b>	132.6	502.07
VIX	0.4234	1.451	0.29	0.771	-2.436	3.283		0.783	1.420	0.55	0.582	-2.016	3.583
Constant	20.34	91.208	0.22	0.824	-159.44	200.13		65.33	89.814	0.73	0.468	-111.7	242.37
R-squared = 0.1450							R-squared = 0.1791						

(B) Bond Residual Spread (SWAP)							(D) CDS Residual Premium (SWAP)						
	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]			Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
CMT 5Y	-11.57	9.768	-1.18	0.237	-30.82	7.68		-12.65	9.797	-1.29	0.198	-31.96	6.661
Default Prem.	<b>0.6152</b>	<b>0.295</b>	<b>2.08</b>	<b>0.038</b>	0.033	1.197		0.156	0.311	0.5	0.615	-0.456	0.770
Swap 5Y	-0.5022	3.45E-01	-1.45E+00	0.147	-1.183	1.79E-01		-0.575	0.366	-1.57	0.118	-1.296	0.146
SP500 Return	<b>336.5</b>	<b>99.034</b>	<b>3.4</b>	<b>0.001</b>	141.31	531.75		<b>286.9</b>	<b>89.507</b>	<b>3.21</b>	<b>0.002</b>	110.51	463.39
VIX	0.5847	1.430	0.41	0.683	-2.235	3.404		0.712	1.380	0.52	0.606	-2.008	3.432
Constant	-0.1171	89.068	0	0.999	-175.69	175.45		56.85	88.238	0.64	0.52	-117.07	230.79
R-squared = 0.2382							R-squared = 0.1577						

TABLE 10: RESIDUAL ANALYSIS (CROSS SECTION)

(A) Bond Residual Spread (CMT)							(C) CDS Residual Premium (CMT)						
	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]			Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
Coupon	14.84	8.859	1.68	0.097	-2.716	32.401		11.580	13.599	0.85	0.396	-15.3733	38.535
Trans Size	-4.51e-07	4.75e-07	-0.95	0.345	-1.39e-06	4.91e-07		-1.02e-06	6.60e-07	-1.54	0.126	-2.33e-0	0.000
Issue Amount	-.00004	.000035	-1.14	0.255	-.00011	.00003		-.000025	.00005	-0.51	0.610	-.000125	0.000
Firm Size	.00009	.0002	0.44	0.662	-.0003	.00049		.000153	.00029	0.53	0.596	-.000419	0.001
Bond Maturity	.5522	3.155	0.18	0.861	-5.7012	6.805		2.9038	3.582	0.81	0.419	-4.1964	10.004
On-the-run	66.448	57.115	1.16	0.247	-46.752	179.648		78.7305	68.619	1.15	0.254	-57.270	214.732
Constant	-41.961	88.585	-0.47	0.637	-217.53	133.61		-112.393	120.99	-0.93	0.355	-352.21	127.424
R-squared = 0.0189							R-squared = 0.0227						
(B) Bond Residual Spread (SWAP)							(D) CDS Residual Premium (SWAP)						
	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]			Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
Coupon	12.741	9.0080	1.41	0.160	-5.11	30.595		13.713	13.357	1.03	0.307	-12.759	40.187
Trans Size	-5.04e-07	4.71e-07	-1.07	0.287	-1.44e-06	4.30e-07		-9.73e-07	6.50e-07	-1.50	0.137	-2.26e-06	0.000
Issue Amount	-.00004	.000035	-1.18	0.242	-.00011	.000029		-.00002	.000048	-0.45	0.653	-.000119	0.000
Firm Size	.00009	.00021	0.41	0.679	-.0003	.00050		.00012	.00028	0.44	0.660	-.00044	0.001
Bond Maturity	2.0089	3.1926	0.63	0.531	-4.319	8.337		3.0564	3.607	0.85	0.399	-4.0927	10.206
On-the-run	59.847	56.149	1.07	0.289	-51.438	171.13		87.900	65.156	1.35	0.180	-41.237	217.038
Constant	-93.394	89.442	-1.04	0.299	-270.66	83.87		-129.073	120.27	-1.07	0.286	-367.456	109.308
R-squared = 0.0231							R-squared = 0.0258						



**Figure 1: Summary view of estimation output.** The uppermost left panel plots the weekly averaged asset volatility across firms. The remaining panels plot the asset value, firm leverage and market capitalizations respectively.



**Figure 3: Difference between Bond and CDS residuals.** The upper panel plots this difference for CMT based residuals, in comparison with 5 and 10 year swap spreads. The lower panel plots the same measure based on the swap curve.

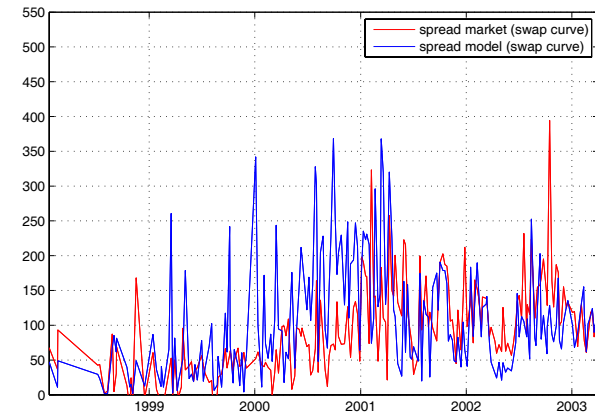
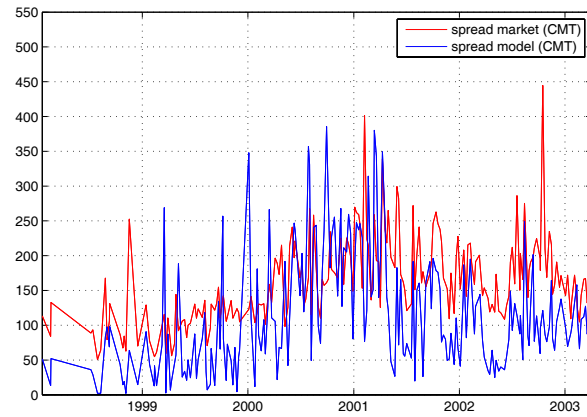
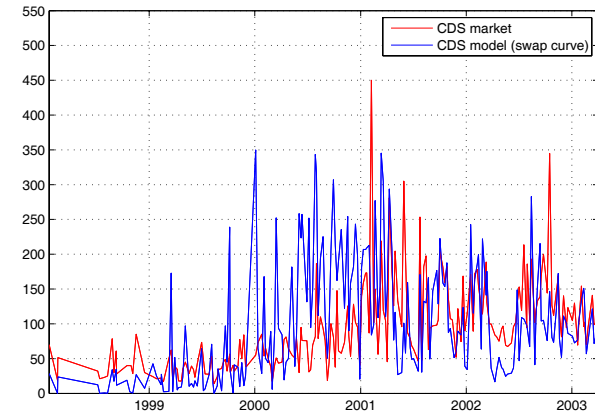
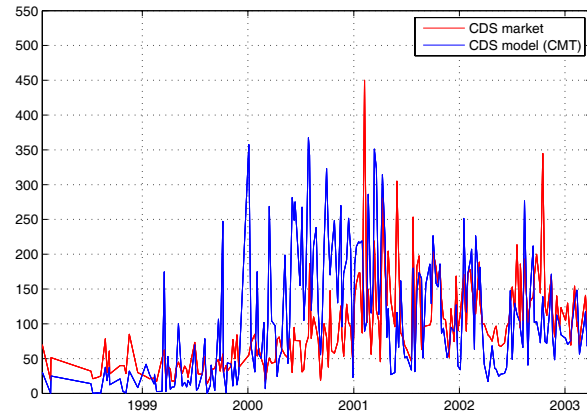


Figure 1: **Figure 2: Summary of pricing output.** The upper (lower) two panels represent results for CDS (bond) pricing. The left panels are based on the CMT curve as benchmarks whereas the right ones use the swap curve.