Estimating the Loads and Energy Yield of Arrays of Wave Energy Converters under Realistic Seas

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Abstract

Estimates regarding the assessment of the energy absorption characteristics of an array of wave energy converters (also referred to as a wave farm) are presented. Regular and irregular waves are used as input in a frequency domain hydrodynamic model which allows iterations in the array layout and farm control strategy. Under such an approach each array element can be controlled independently while keeping the design objective (maximisation of the wave farm energy yield). The distribution of power take-off (PTO) loading on the various array elements, as induced by the incoming sea, is also investigated. The approach is verified by comparing the estimates with results from a semi-analytic method developed at the University of Oxford.

The overall objective of the study is to quantify the influence of the array layout and farm control in the performance of a wave farm under the action of irregular waves. The results show that the energy yield and the PTO loads are affected by such factors; hence these can be seen as key design drivers in order to reduce the uncertainty and thus the cost of energy when planning a wave farm. Further studies may address additional constraints, either technical or economical. This study is expected to contribute to the development of specific modules of a design optimisation tool for wave farms, and extends the findings originally presented at the 8th European Wave and Tidal Energy Conference.

1 Introduction

The commercial development of wave energy in utility-scale projects will rely on the deployment of multiple units in an array, creating multi-MW wave farms. Several engineering challenges are associated with the definition and optimisation of a wave farm, namely the geometrical layout, electrical connections between array elements (and the link to shore), mooring configurations (if applicable), control aspects and hydrodynamic interactions between devices.

All of the above mentioned factors will have an impact on the energy yield. It is therefore essential that project developers have access to tools which are able to accurately assess and optimise the key factors which define the wave farm, maximising the energy output. Such tools should be seen as vehicles which allow project developers and / or investors to assess the suitability and potential of a given technology to a specific project, in a close parallel with the project development standard procedures which are available in wind energy.

Array interactions have been a recurrent R&D topic in offshore engineering over the last decades. Pioneering work has also been done in the wave energy field [e.g. 1-6], focusing primarily on linear arrays of point absorbers, equally and unequally spaced. In the following paragraphs of this section a short review of some selected publications is presented.

In [7] an overview of the key results related to the performance of wave energy converters when placed in arrays is presented, along with the fundamental principles behind some hydrodynamic interaction theories used in the field, such as the point absorber and the plane wave approximations. The latter are also approached in more detail in [8]. In summary the point absorber assumption neglects the scattered wave field, which is equivalent to assuming that $ka \ll 1$ (where k is the wavenumber and a is a length scale such as the radius of a floating device). The plane wave approximation can be summarised as a wide spacing assumption, resulting in a wave field comprising plane waves incident on each element in the array. In [7] regular waves results for small arrays of heaving or surging buoys address the interaction factor q, i.e. the ratio between the power absorbed by the array and the N times the power absorbed by an isolated wave energy converter (WEC, here also referred to as a device or element), where N is the number of elements of the array; values of q are presented as a function of the nondimensional spacing kd between array elements, for several wave headings β. The results illustrate the potential for constructive and destructive interference, i.e. the existence of wave frequencies for which the array arrangement is beneficial $(q > 1)$ or detrimental $(q < 1)$ with regard to farm energy yield. Although early work shows that a net gain may still be possible, in [6] it is mentioned that a practical strategy for the design of wave farms may involve the minimisation of the destructive interference effects. Due to these effects the necessity of a tool to accurately quantify accurately the energy output of a wave farm becomes clear, as a means to reduce the uncertainty associated with the project development stage if commercial wave energy projects are to be considered

The critical factors that condition the energy output and thus the q factor need to be identified and their influence quantified. Two that have been previously addressed under the action of regular waves are control [9] and layout [e.g. 10], which in practice will be conditioned by device access routes, mooring layout (if applicable) and overall footprint of the wave farm (linked with the number of devices and thus the power rating of the farm). The previously mentioned tool will need to be able to quantify the influence of all of these, plus additional factors such as real seas, device geometry and modes of motion linked with the wave power absorption (PTO modes). In a much more reduced number of references irregular waves have been used as input. In [11] the authors report, for an array of twenty heaving point absorbers, a net gain of up to 20% in the energy absorption by using phase control. The influence of incident wave angle is also assessed.

In this paper a preliminary study is presented, which addresses the influence of control and layout on the wave farm energy yield under the action of accurate descriptions of realistic seas. The analysis of the response to different irregular wave patterns should be seen as a critical design factor which influences the design. Firstly comparisons with results from a semi-analytical method for an uncontrolled array of truncated cylinders previously presented in [12] are made, in order to verify the solution of the radiation and diffraction problems, with the latter allowing also a discussion related to the excitation forces associated with each array element. The optimal q for the array of truncated cylinders specified in [13] is then presented as a function of ka and β, providing a benchmark and a starting point when extending the study to irregular waves. The findings of this paper are expected to contribute to the creation of a tool that will assess, iterate and optimise the design of wave farms of N elements, each being of arbitrary shape and with an arbitrary number of bodies.

2 Motivation and Approach

The potential for constructive interference (i.e. the possibility of achieving an energy yield with a farm of N elements which exceeds the output of N WECs working in isolation) has long been identified. Equally identified is the fact that it is not realistic to assume that such constructive behaviour will be present for the full range of incident wave frequencies, thus there will also be destructive interference effects. The objective function when designing a wave farm is therefore twofold: 1) maximise the effects of constructive interference; 2) minimise the effects of destructive interference.

The motivation for this preliminary study is to assess the influence of several critical variables (wave climate, layout and control strategy) in the design of a wave farm. To verify the approach, which is based in the frequency domain and uses the solutions of the radiation (hydrodynamic coefficients) and diffraction (exciting forces) problems provided by a Boundary Element Method (BEM) solver (WAMIT), comparisons with semi-analytic methods [12] and [13] are firstly made for an array of uncontrolled devices. The same approach is then extended to a controlled array, in which each individual element can be independently controlled via an external damping term associated with the relative heave motion. The energy absorption results are a function of the incoming sea state.

The array configuration under study is shown schematically in Figure 1. It consists of a square array of four cylinders with radius a and draft $2a$, with centres equally spaced by $4a$ (additional results for a spacing of $6a$ were calculated when using irregular waves as input). The water depth is equal to $4a$. The regular wave results use a =1m, as in [12] and [13], whereas the irregular wave results use a = 10m, providing results which are more meaningful for wave energy conversion.

Linear wave theory is applied throughout this paper. As a result there are some well defined simplifying assumptions such as:

1. The free-surface and the body boundary conditions are linearised;
2. The fluid is incompressible and the flow is irrotational (potential flow): $\nabla^2\Phi = 0$, where Φ is the velocity potential;
3. Viscous effects like shear stresses and flow separation are not considered;
4. Under these above mentioned assumptions all variables can be expressed as a complex amplitude multiplied by $e^{i\omega t}$ (regular waves, sinusoidal motions).

The velocity potential can therefore be given by

$$\Phi = \mathrm{Re}\left\{\phi e^{i\omega t}\right\}, \qquad (1)$$

where ϕ is the complex velocity potential, Re denotes the real part, ω is the angular frequency of the incident wave and t is time. By assuming the linear decomposition of the problem, the velocity potential can be obtained as the sum of the radiation and the wave exciting components,

$$\phi = \phi_R + \phi_S , \qquad (2)$$

where ϕ_R is the radiation potential and ϕ_S the scattered potential, respectively given by

$$\phi_R = \sum_{j=1}^{6MN} \xi_j \phi_j , \qquad (3)$$

and

$$\phi_S = \phi_0 + \phi_D , \qquad (4)$$

where ϕ_0 is the velocity potential associated with the incident (incoming) waves and ϕ_D the diffraction potential. In Eq. (3) ξ_j are the complex amplitudes of oscillation in all the available degrees-of-freedom (6*MN*) and ϕ_j the corresponding unit-amplitude radiation potentials (those resulting from the body motion in the absence of an incident wave). Note that the summation is limited to 6*MN*, where *M* is the number of bodies per WEC and *N* is the number of WECs in the array. In the present case each array element has just one body and *N*=4, thus 6*MN* = 24. Even in cases where $M > 1$ it is unlikely that all 6*M* degrees-of-freedom are available, i.e. a multi-body WEC will necessary have locked modes, with power being absorbed in one or two relative modes of motion (and thus a total of 7 or 8 degrees-of-freedom). It is important that the tools developed are able to cope with locked modes to reduce the computational effort while being flexible to allow all possible combination of locked modes.

The complex amplitudes of oscillation ξ_j are given by

$$\sum_{j=1}^{6MN}\left[-\omega^2\left(M_{ij}+M_{ij}^E+A_{ij}\right)+i\omega\left(B_{ij}+B_{ij}^E\right)+\left(C_{ij}+C_{ij}^E\right)\xi_j\right]=X_i\,, \qquad (5)$$

where M_{ij} is the mass matrix, M_{ij}^E, B_{ij}^E and C_{ij}^E the externally applied mass, damping and stiffness matrices (respectively), A_{ij} is the added-mass matrix, B_{ij} is the radiation damping matrix, C_{ij} is the hydrostatic stiffness matrix and X_i the exciting force in a plane wave of unit amplitude. The phase is defined such that the crest of the undisturbed wave would be at the centre of the array. Note that in this preliminary study M_{ij}^E and C_{ij}^E are everywhere null (i.e. inertia tuning by changing the mass of systems with e.g. ballast tanks is not considered and there is no externally applied stiffness component via e.g. a mooring line). In the numerical solution, A_{ij}, B_{ij} and X_i are calculated using WAMIT.

The absorbed power and associated control of the array and its elements are therefore limited to the influence of B_{ij}^E. In this preliminary approach the only elements of B_{ij}^E which are not null are the diagonal terms associated with the heave motion of each of the four array elements ($B_{33}^E, B_{99}^E, B_{15\ 15}^E, B_{21\ 21}^E$). The farm control strategy therefore involves four independent external damping terms. By normalising the complex amplitudes of oscillation ξ_j with the wave amplitude *A*, leading to $\overline{\xi}_j=\xi_j/A$, it is possible to derive an auxiliary absorber power P_{aux} in each element, *n*, which is given by

$$P_{aux}=\frac{1}{2}B_{ij}^E\omega^2\left|\overline{\xi}_j\right|^2, \qquad (6)$$

which has dimensions of W/m^2 (and here i=j=6(n-1)+3). The usefulness of P_{aux} is particularly clear when evaluating the average absorbed power per farm element, $\overline{P}_n$, under the action of irregular waves (using the superposition principle). By definition the frequency spectrum $S(f)$ of an incident irregular wave can be expressed by (note that $A(f)$ is the wave amplitude spectrum)

$$S(f) = \frac{1}{2}\frac{A^2(f)}{df}, \qquad (7)$$

and thus $\overline{P}_n$ can be given by

$$\overline{P}_n = \int 2P_{aux}S(f)\,df\,. \qquad (8)$$

Finally, the average absorbed power by an array can be quantified by

$$\overline{P} = \sum_n^N \overline{P}_n\,. \qquad (9)$$

Note that in the irregular wave case considered here, B_{ij}^E is unchanged throughout the frequency range for each $S(f)$. The superposition principle can be applied as the system remains unchanged from frequency to frequency. It is also valid if B_{ij}^E is a function of the frequency, an analysis that can be done in future work. In practice this corresponds to a (conservative) control strategy that aims to tune the response to a particular sea state, described by $S(f)$. The objective function is the maximisation of $\overline{P}$, under all the previously mentioned constraints. In this paper the assessment is limited to the influence of the sea state, array geometry and control strategy.

The next section is divided into the several stages leading to a preliminary assessment of relative influences of the various parameters listed above, related to the design of a generic wave farm of four cylindrical heaving WECs. The first results presented are associated with the verification of the approach for solving the radiation and diffraction problems, ensuring that the BEM method applied is consistent with the semi-analytical models. Secondly, optimal results for regular waves over a range of wave headings are presented for heaving cylinders; such results allow comparisons with the subsequent results which are related to irregular waves. In the third stage a sensitivity analysis related to the damping coefficient as a function of the incident frequency spectrum for an isolated cylinder is presented. Lastly, arrays of four cylinders (the square layout described in Figure 1) are assessed. In this analysis each of the four cylinders is controlled via an independent damping coefficient (thus a total of four terms), and the average power absorbed by the array $\bar{P}$ is evaluated. Hence for each sea state an assessment is made of the coupled influences of array layout and different farm control strategies (here consisting of a layer of four independent variables, namely the damping coefficients).

3 Results

3.1 Comparison with Analytical Solutions

The first set of results is presented in Figure 2, where the solutions of the diffraction and radiation problems derived analytically in [12] and [13] are compared with new BEM predictions. The configuration under analysis is the array described in Figure 1. The comparisons are limited to the exciting force (X_i) and the hydrodynamic coefficients (A_{ij} and B_{ij}). With regard to the latter, the comparisons use representative terms of the real and the imaginary part of radiation impedance matrix *Z*, respectively equal to B_{ij} and ωA_{ij} . As mentioned at the end of the paragraph containing Eq. (5), A_{ij} , B_{ij} and X_i are the only variables directly obtained from the BEM solution (the equation of motion is solved in a separate code, to ensure that controlled multi-body WECs can be conveniently modelled).

The objective of these initial comparisons is to verify the approach, ensuring its applicability to the subsequent steps, namely when solving the equation of motion and adding the farm control layer(s). Figure 2a shows comparisons of the modulus and phase of the wave exciting force (*x*, *y*, *z* components) for $\beta = 0$ (where the angle β is defined in Figure 1, and phase is defined as after Eq. (5)). The four cylinders are identified by the numbering listed in Figure 1, and the BEM results are marked with the 'WAMIT' legend. Overall the correlation between the analytical and the numerical results is very high, with most of the discrepancies occurring for low values of *ka*, and thus are more significant for longer waves and small WECs (i.e. small values of *a*). Results for other incident wave directions allow similar conclusions. With regard to the radiation impedance matrix, which is [24 x 24], Figure 2b limits the comparisons to some typical cross-coupling terms. Minor discrepancies between the analytical and numerical results are once again seen for low values of *ka*.

Results for complex geometries and multi-body WECs are more difficult to compare, as analytical solutions for such configurations are harder to derive. The usefulness of the BEM solution is clear in such cases, albeit it is important to ensure that adequate comparisons with experimental results (validation) are made, particularly if comparison with analytical results (verification) is not possible.

3.2 Interaction Factor for Regular Waves

In the vast majority of the studies available in the literature [e.g. 1-10], the assessment of the performance of an array of WECs has been approached under the action of regular waves. Even with significant limitations (quite often the peak values of the interaction factor q are obtained under unrealistic amplitudes of motion) such an approach is still useful for assessing the main constructive and destructive interference areas and the wave headings that most contribute to constructive effects. Nevertheless it is important to emphasise that conclusions from regular wave simulations will always be limited, and the following sections of the present paper address more realistic conditions (namely the influence of irregular waves and suboptimal control settings).

In this section the interaction factor q was calculated as a function of the nondimensional wave frequency ka and the wave heading β. In Figure 3 the cylinders can absorb energy in heave. In [14] an additional energy absorption mode (surge) was also analysed.

In Figure 3a the interaction factor is shown as a surface, plotted against dimensionless wavenumber and the wave heading. The row of peaks along $\beta = \pi / 4$, shown as a cross-section in Figure 3b, corresponds to the array being oriented diagonally.Near trapping can be clearly observed at certain wavenumbers. The central peak corresponds to q = 2.3, i.e. the array of four cylinders is absorbing about nine times what a single cylinder of the same dimensions could at this wavenumber. The equivalent cross-section for $\beta = 0$ is also shown in Figure 3b.

Although the regular wave investigation only allows limited conclusions, its usefulness is clear in Figure 3: such assessment of q leads to the identification of areas and scenarios which may be starting points for further, more refined, studies which address e.g. the effects of irregular waves. A preliminary study (limited to absorption in heave) concerning the influence of representative seas on the energy yield from the array is presented in the following sections. Particular attention is given to the $\beta = \pm\pi / 4$ case, which can be physically associated with either a change in the dominant direction of the incoming sea or a change in the layout of the array.

3.3 Suboptimal Damping Settings for Irregular Waves – Single WEC

In order to design a wave farm a realistic representation of the sea state needs to be used as input to the simulation. A preliminary assessment of the influence of this additional factor is done in the following section, but first it is relevant to quantify the power absorption characteristics of a single WEC (acting in isolation) under the same irregular waves in order to compare its response with that of each element in an array.

In this study the influence of different spectra (and different incident wave directions, in the array layout) was assessed. Two spectral parameters, the significant wave height H_{m0} and the energy period T_{-10}, plus the shape of the frequency spectrum (Bretschneider or JONSWAP), influence the results and thus these parameters were used to define the irregular wave input. Table 1 summarises the frequency spectra that were assessed. In some cases the peak period T_{peak} is specified, and its value set equal to the period associated with the peak capture width (a regular wave result). Such assessment tries to evaluate the effect of the incoming sea in the device response, which given the simplicity of the design is narrow banded with regard to wave frequency.

Note that the different wave directions β are irrelevant in the single body case, as the cylinder is axisymmetric. Table 1 applies also for the array simulations under irregular waves (see the following section), hence the inclusion of β. Further studies can consider different modulated spectral shapes or real (measured) seas.

Rather than searching for the absolute optimum with regard to energy capture under the influence of the sea states described in Table 1 (recall that the optimal solution is typically associated with large and unrealistic amplitudes of motion), suboptimal solutions following a conservative control approach were evaluated. This was implemented by defining the PTO force as the product of an external damping coefficient (D_{ext}) and velocity, and by iterating on the value of D_{ext} for each WEC.. It is recognised from inception that such a constrained optimisation exercise will not result in the absolute maximum in terms of energy absorption, yet it will result in an achievable estimate.

The cylinder is controlled via a single control term (i.e. the external damping coefficient, D_{ext}), allowing an assessment of the sensitivity of the solution to variations in the control setting under the action of realistic seas. In the case of the array layout, each cylinder is controlled independently (hence the farm control matrix has four independent terms). To obtain more representative dimensional results the radius of the cylinder a was altered to 10m, and the other geometrical properties listed in Figure 1 apply unless otherwise stated. The single cylinder iterations provide the benchmark for the calculation of the interaction factor q under the influence of irregular waves.

Furthermore, it is interesting to record not only the value of the maximum absorbed power, averaged over the frequency spectra specified in Table 1 (calculated from Eq. (8) for a single WEC), but also the damping coefficient D_{ext} associated with such a situation. This allows comparisons with the individual damping coefficients associated with each array element when deriving the maximum power absorbed by the farm.

Table 2 summarises the maximum power and the associated damping coefficient for the six cases identified in Table 1. When comparing cases with the same H_{m0} and spectral shape (cases 2 and 4) the influence of the wave period in D_{ext} is clear. Cases 3 and 5 also demonstrate the effect of de-tuning for low values of T_{peak} (as the $Max\ \overline{P}_n$ values obtained are low, such cases will not be considered in the subsequent Sections of this paper). The influence of the spectral shape is also clear when comparing cases 4 and 6 (note that in this example the magnitude of such effect is mostly related to the narrow banded response of the WECs; see also Figure 4). Finally, recall that under the linear approximation all power results are proportional to H_{m0}^2 (for the same wave period and spectral shape).

Figure 4 illustrates the power absorption characteristics of the heaving cylinder, as a function of the wave period and external damping coefficient for two selected sea states. Note that the absorbed power is presented with dimensions of MW/Hz, corresponding to the absorbed power associated with each monochromatic component, which is then integrated over the entire period range allowing the quantification of the averaged absorbed power per damping coefficient (and thus the identification of the maximum absorbed power per sea state). The influence of the natural period in heave is clear in Figure 4 and associated with the peak response in both cases (wave period equal to 10.2s).

As mentioned above Table 2 shows the narrow frequency response of the heaving cylinder, illustrating the suitability of the WEC to the longer peak period as expected from its capture width characteristics. The wave period dictates the external damping coefficients D_{ext} associated with the maximum absorbed power for each sea state. The influence of the spectral shape can be quantified: although case 6 has the same spectral parameters as case 4, the change from Bretschneider to JONSWAP (thus a narrower spectrum) results in the increase of the absorber power by a factor of 2.53 (this is also linked with the narrow capture width properties of such geometry). Finally, note that the peak power (in MW/Hz) is not necessarily associated with the maximum absorbed power (in kW) over the entire sea, and thus the two events can (and for the cases listed in Table 1 do) occur for different damping settings.

These results are particularly relevant for comparisons between the array output and the output of single cylinder times *N* (number of WECs). Also, the external damping values that were obtained can be seen as a starting point for the farm control strategy iteration, which involves four independent damping terms, associated with each array element.

3.4 Suboptimal Damping Settings for Irregular Waves – Arrays of WECs

This section presents a preliminary assessment of the combined influences of the incoming sea state, array layout and control strategy in the wave farm energy yield. Ultimately, it is expected that such studies can benefit the creation of a software package which is able to quantify the interaction effects, iterate and optimise the design of the wave farm, taking these and other operational constraints into account (e.g. electrical cable length, mooring strategy, minimum / maximum distance between each WEC, etc.).

In this preliminary assessment three aspects are particularly critical: 1) the influence of the layout of the array on the energy yield under the action of irregular waves; 2) the potential to further improve the energy yield by adjusting the horizontal position and control of each farm element (farm control strategy); 3) the quantification of the interaction factor q for irregular waves in the above mentioned scenarios. As in the previous sections a suboptimal solution was undertaken (based on the one term control strategy per WEC through the external damping coefficient D_{ext}), in order to obtain more conservative estimates. All the geometrical properties of the array are listed in Figure 1, although as for the isolated cylinder case the radius a was altered to 10m. Note also that two different spacings were analysed (4a and 6a). The study was focused on the square array of four elements, the influences of the sea states described in Table 1, the farm control strategy and two dominant wave directions ($\beta = -\pi / 2$ and $\beta = -\pi / 4$). The latter case corresponds physically to the array being oriented diagonally (hence a layout change).

Tables 3 and 4 summarise the key results: the maximum averaged absorbed power $\bar{P}$ and the interaction factor q. The suboptimal farm control strategy iterates on each of the four external damping coefficients which control the four array elements with step changes of 10kNs/m.

It should be emphasised that, as in the isolated cylinder case, the control strategy that was implemented is suboptimal, as a single external damping value is implemented throughout the sea state on each WEC (passive control per sea). A more complex control strategy would involve changing this control term for each incoming wave period or for each representative wave group (active control). Further complexity is introduced by controlling more than just the external damping (e.g. external stiffness) or by absorbing in more than one degree-of-freedom. More complex WECs are likely to involve more than one body, and thus the potential to explore other control and layout configurations is also higher.

This methodology deliberately tries to obtain more realistic estimates than those linked with optimal control by assessing the amplitudes of motion (which are kept small when compared to the wavelength), although it is recognised that improvements can be obtained via the implementation of less simplistic approaches. The main value of such assessment is in the quantification of the combined effects of layout and control iterations under irregular waves in the farm energy yield. As Table 3 shows there is an average increase of 4% due to the layout and control iterations (when changing β from $-\pi/2$ to $-\pi/4$) for a spacing of 4*a* (with the exception of case 2). Such a figure would have a significant impact on the energy yield and thus on the revenue of a wave farm. Table 4 quantifies the influence of an iteration in the array spacing (from 4*a* to 6*a*) for $\beta = -\pi/4$ rad. Case 6 shows an increase of 5% with regard to q factor, which is physically linked with the increase in the power absorbed by cylinder 3 (i.e. the rear cylinder). It is encouraging to acknowledge that such results were derived in a simplified scenario (simple geometry, single body per WEC, one term controller per WEC).

Figure 5 shows the type of quantitative assessment that can be made when evaluating different control approaches for each case. When a wide range of damping settings is analysed, zooming out of such figures allows the assessment of the implications of under- and over-damped scenarios, while zooming in allows the effect of the tuning associated with the four term farm control layer to become clearer. In Figure 5, and due the control settings alone, there is a difference of over 20% in power absorption figures of the wave farm, which emphasises the importance of fully understanding the coupled influences of incoming sea, layout and control strategy.

Finally, it is also interesting to compare the values of the damping coefficients associated with each of the four cylinders and compare such values with the analogous coefficient linked to the isolated cylinder case. For example, for case 6 with $\beta = -\pi/4$ and a spacing of 6*a* the suboptimal damping setting for cylinders 1 to 4 are 430, 350, 690 and 430 kNs/m, which can be compared to the 390 kNs/m (thus 77% smaller than the maximum value applied in the array case). Similar variations are obtained for the other cases listed in Table 3. This range may induce changes in the engineering specifications of the PTO, when considering the implementation of the full range of damping coefficients in a real application.

Under these simplified assumptions the interaction factor q is always below 1 for each of the sea states tested, which places the farm design objective as the minimisation of the effects of destructive interference. Further iterations can include other variables such as more advanced control techniques and / or irregular layouts to potentially mitigate minimise aspects. As mentioned previously, in [11] a net gain (constructive interference) is quoted, which emphasises the need for a tool that accurately predicts the energy yield and quantifies all relevant effects.

4 Conclusions

A preliminary assessment of a simplified WEC array has been conducted, quantifying the effects of layout and control iterations under the action of irregular waves. To increase confidence in the numerical predictions, the fundamental hydrodynamic properties were firstly benchmarked against analytical results, and a regular wave investigation was also carried out to outline the parameter space of interest and indentify the most promising layouts which should be the subject of more realistic assessments using realistic seas. The PTO settings for a WEC working in isolation were also derived to provide not only a benchmark but also a starting point for each farm element PTO setting. The ultimate objective of this approach is the creation of a software tool that assesses and quantifies the interaction between the several array elements, optimising the output. Particular attention is given to forces (wave induced and externally applied) and the power absorption characteristics of the wave farm, allowing comparisons with N times the output of an isolated WEC (N being the number of WECs in the array).

For simplicity the study was focused on a simple geometry (circular cylinder), a limited (and fixed) number of WECs (four), and a suboptimal control approach which allowed only one term to be controlled per WEC (external applied damping in heave). Even in this simplified scenario, layout and control iterations led to an average increase of 4% in the farm's energy absorption properties, a figure that would have a significant impact in the energy yield. Furthermore, the suboptimal nature of the control strategy (deliberately tested to ensure that conservative estimates were obtained) limited the potential of the approach. For example the individual damping coefficients are kept fixed for each sea state (passive strategy, suited to tune a WEC in a statistical sense, i.e. for the duration over which the sea state is characterised). A more complex methodology would involve changing these settings in accordance with the measurement of the incoming wave time history (active strategy). Following a similar technique a net gain (constructive interference) is quoted in [11] using irregular waves as input, which emphasises the need for a tool that accurately predicts the energy yield and quantifies all relevant effects.

Further studies can address (among other parameters) variations in the number of WECs (farm installed capacity), WEC spacing and irregular layout (which is limited for operational and non-technical reasons), and other externally applied forces (e.g. mooring loads and nonlinear mechanical loads as in [15]). The implementation of alternative algorithms such as those described in [8] and [16] may also provide a means to reduce the computational effort associated with the design of a wave farm. Other recent studies (e.g. [17]) have reached similar conclusions in particular with regard to the influence of increasing element spacing in the power absorption characteristics of the array.

References

[1] Budal, K. Theory for absorption of wave power by a system of interacting bodies. *Journal of Ship Research*, Vol. 21, 248-253, 1977.

[2] Falnes, J. and Budal, K. Wave-power conversion by point absorbers. *Norwegian Maritime Research*, Vol. 6, 2-11, 1978.

[3] Evans, D. V. Some theoretical aspects of three-dimensional wave-energy absorbers. *Proc. First Symposium on Wave Energy Utilization*, Gothenburg, Sweden, pp. 77-113, 1979.

[4] Falnes, J. Radiation impedance matrix and optimum power absorption for interacting oscillators in surface waves. *Applied Ocean Research*, Vol. 2, 75-80, 1980.

[5] McIver, P. Some hydrodynamic aspects of arrays of wave-energy devices. *Applied Ocean Research*, Vol. 16, 61-69, 1994.

[6] Thomas, G. and Evans, D. V. Arrays of three-dimensional wave-energy absorbers. *Journal of Fluid Mechanics*, Vol. 108, 67-88, 1981.

[7] McIver, P. The hydrodynamics of arrays of wave-energy devices. *Wave Energy Converters – Generic Technical Evaluation Study (Annex B1)*, Report to the Commission of European Communities, 1993.

[8] Mavrakos, S. and McIver, P. Comparison of methods for computing hydrodynamic characteristics of arrays of wave power devices. *Applied Ocean Research*, Vol. 19, 283-291, 1998.

[9] Justino, P. and Clément, A. Hydrodynamic performance of small arrays of submerged spheres. *Proc. 5th European Wave Energy Conference*, pp. 266-273, 2003.

[10] Fitzgerald, C. and Thomas, G. A preliminary study on the optimal formation of an array of wave power devices. *Proc. 7th European Wave and Tidal Energy Conference*, 2007.

[11] Budal, K. and Falnes, J. Wave-power conversion by point absorbers. A Norwegian project. *International Journal of Ambient Energy*, Vol. 3, No 2, 59-67, 1982.

[12] Siddorn, P. and Eatock Taylor, R. Diffraction and independent radiation by an array of floating cylinders. *Ocean Engineering*, Vol. 35, 1289-1303, 2008.

[13] Yilmaz, O. and Incecik, A. Analytical solutions of the diffraction problem of a group of truncated cylinders. *Ocean Engineering*, Vol. 25, 385-394, 1998.

[14] Siddorn, P. Wave energy absorption by arrays of oscillating bodies. MEng Final Report, University of Oxford, 2007,

[15] Durand, M., Babarit, B., Pettinotti, B., Quillard, O., Toularastel, J.L. and Clément, A.H. Experimental validation of the performance of the SEAREV wave energy converter with Real Time Latching Control. *Proc. 7th European Wave and Tidal Energy Conference*, 2007.

[16] Li, Y. and Mei C. C. Multiple resonant scattering of water waves by a two-dimensional array of vertical cylinders: linear aspects. *Physical Review*, Vol. 76, Article Number 016302, Part 2, 2007.

[17] Babarit, A., Borgarino, B., Ferrant, P. and Clément, A.H. Assessment of the influence of the distance between two wave energy converters on the energy production. *Proc. 8th European Wave and Tidal Energy Conference*, pp. 528-537, 2009.

Formula / shape	*Case Id.*	H_{m0} *[m]*	T_{-10} *[s]*	T_{peak} *[s]*	β *[rad]*
Bretschneider	1	2	7		$-\pi/2$, $-\pi/4$
	2	4	11		
	3	2		6	
	4	4		10.2	
JONSWAP	5	2		6	
	6	4		10.2	

Table 1: Definition of the cases studied and the associated input frequency spectra; the several cases allow the quantification of the influence of key parameters such as the significant wave height, the wave period, the wave direction and the shape of the spectrum; note that the peak enhancement factor (γ) for all JONSWAP spectra was set at 3.3, and that under the linear approximation all power results are proportional to H_{m0}^2 (for the same wave period and spectral shape)

Case Id.	*Max* $\overline{P}_n$ *[kW]*	D_{ext} *[kNs/m]*
1	39.2	590
2	473.2	1140
3	1.6	1900
4	414.1	640
5	0.5	1900
6	1049.7	390

Table 2: Maximum absorbed power (per sea state) by an isolated cylinder absorbing in heave with an external damping coefficient D_{ext}

Case Id.	β *[rad]*	*Max* $\bar{P}$ *[kW]*	q
1	$-\pi/2$	146.6	0.93
1	$-\pi/4$	153.4	0.97
2	$-\pi/2$	1796.6	0.95
2	$-\pi/4$	1825.6	0.96
4	$-\pi/2$	1520.5	0.92
4	$-\pi/4$	1571.8	0.95
6	$-\pi/2$	3680.1	0.88
6	$-\pi/4$	3883.6	0.93

Table 3: Wave farm absorbed power and interaction factor (irregular waves) under a suboptimal control strategy (four independent PTO damping coefficients, one per WEC); WEC spacing = 4a

Case Id.	*Max* $\overline{P}$ *[kW]*	q
1	145.2	0.93
2	1819.1	0.96
4	1569.0	0.95
6	4082.7	0.97

Table 4: Wave farm absorbed power and interaction factor (irregular waves) under a suboptimal control strategy (four independent PTO damping coefficients, one per WEC); WEC spacing = 6a; $\beta = -\pi / 4$ rad

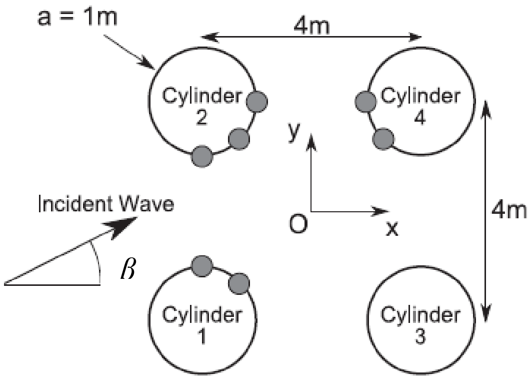

Figure 1: Initial array configuration (from [12]).

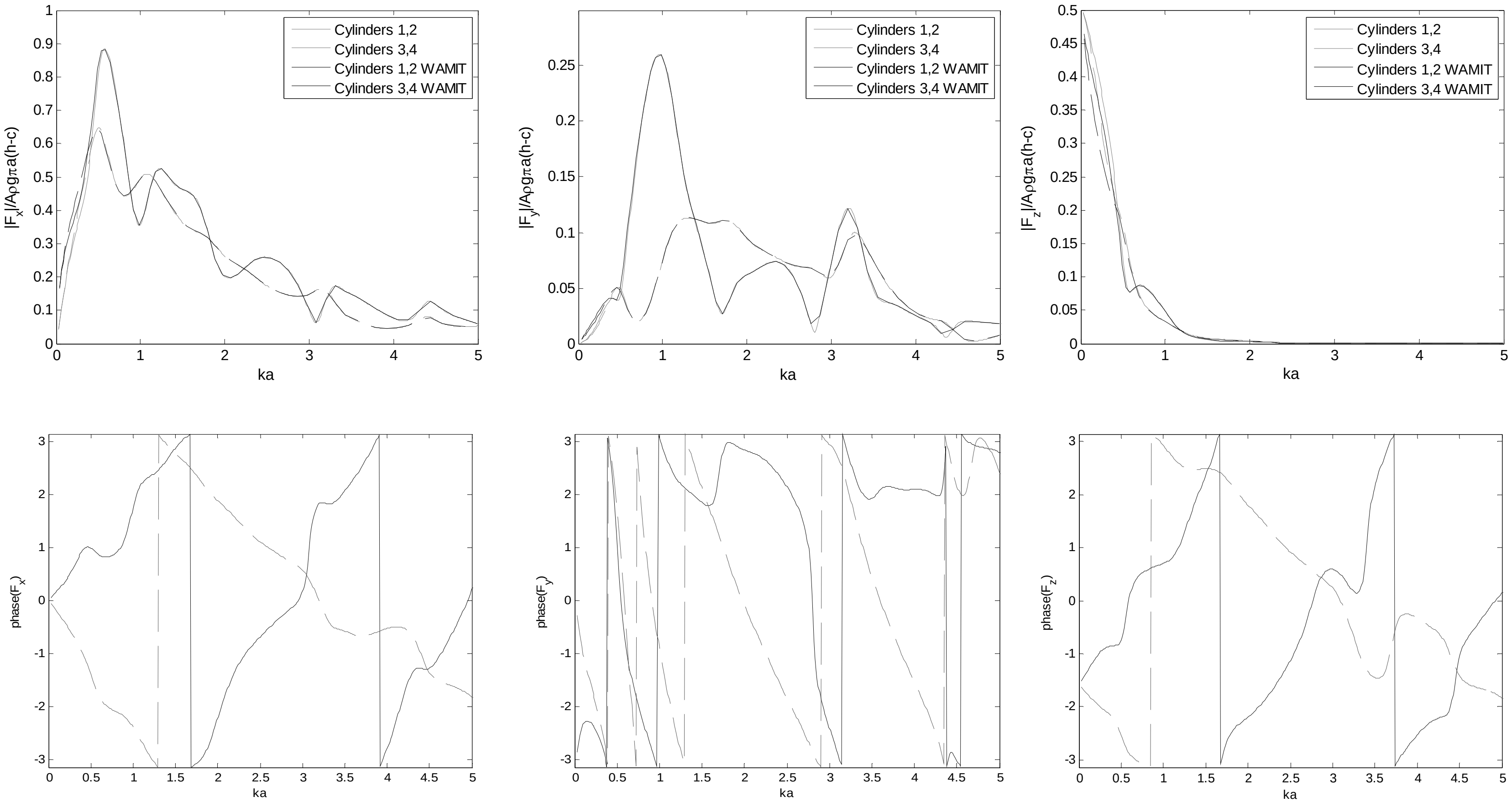

Figure 2a: Comparisons with the wave exciting force (modulus: top row; phase: bottom row) results derived in [12] for $\beta = 0$
Note that the solid lines refer to cylinders 1 and 2 while the dashed lines refer to cylinder 3 and 4.
The differences between the analytical and numerical approach (WAMIT) are mostly visible for low *ka* values in the top row.

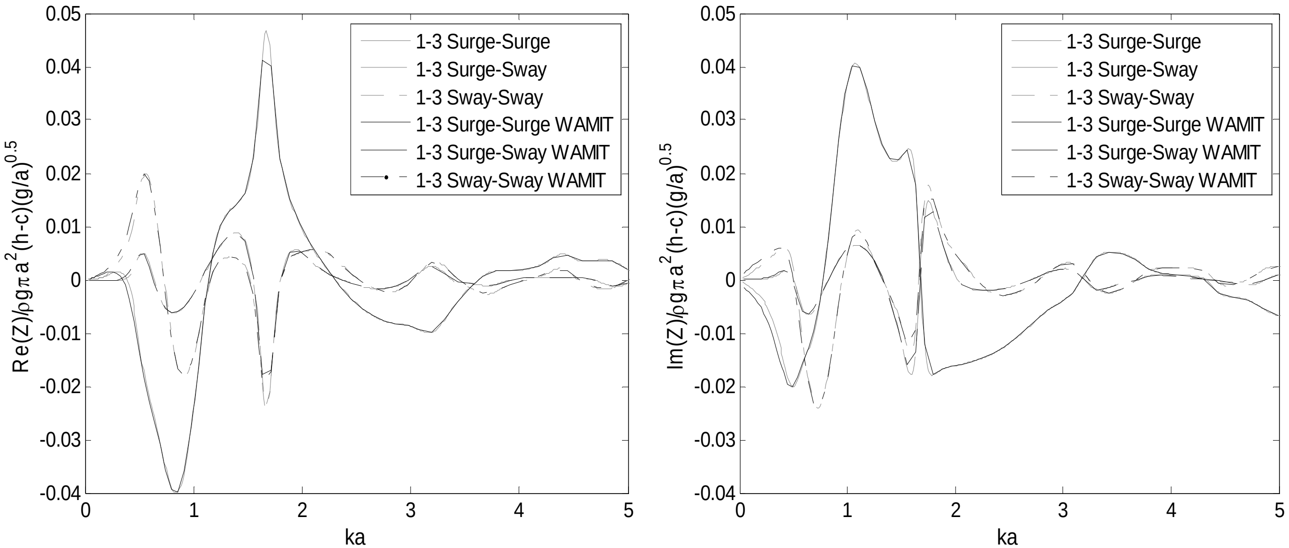

Figure 2b: Comparisons with the radiation impedance results derived in [12] for $\beta = 0$
(surge – sway cross coupling terms)

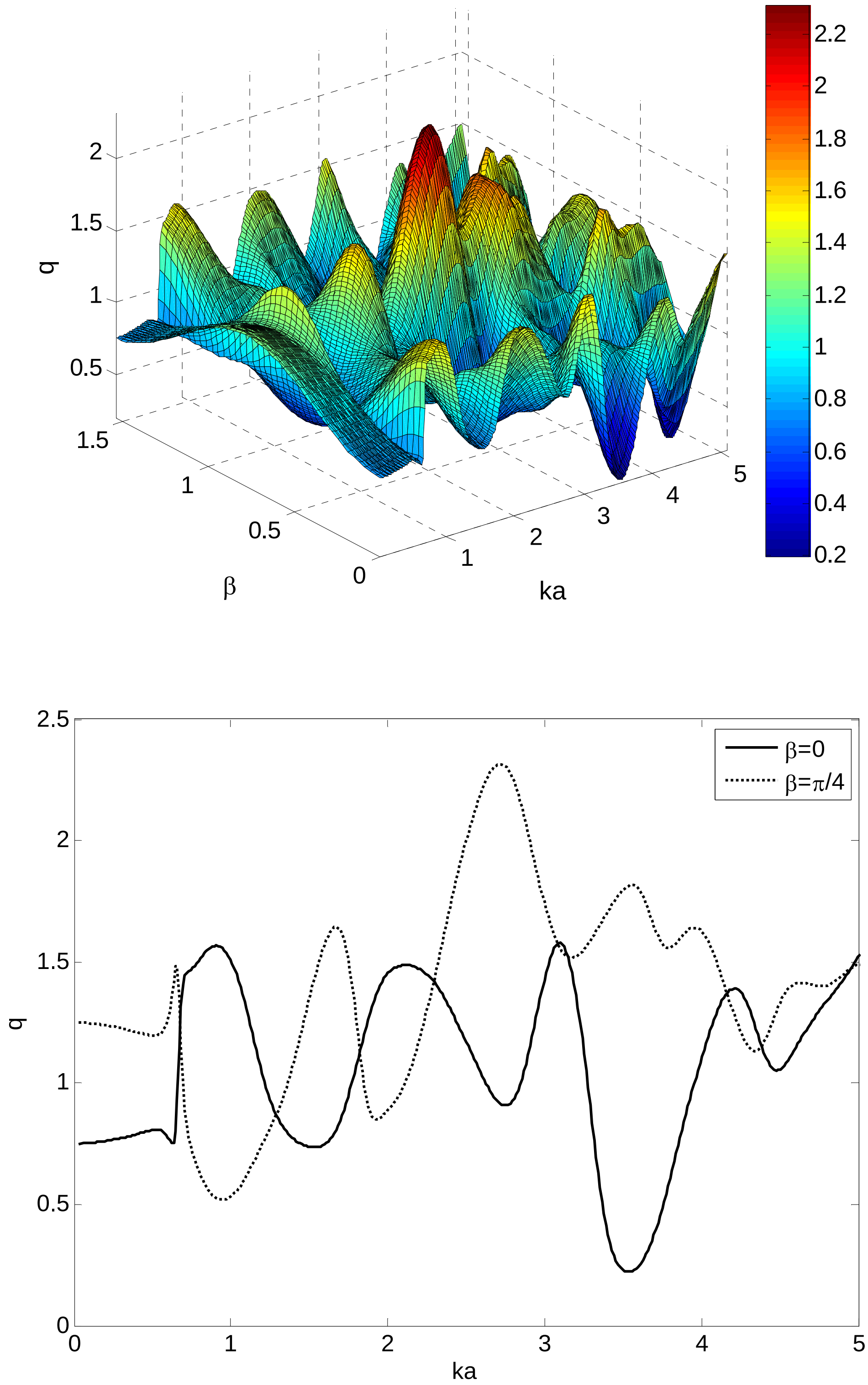

Figure 3: a (top) - Interaction factor q for cylinders absorbing in heave;

b (bottom) - Evolution of q with ka for selected wave angles β.

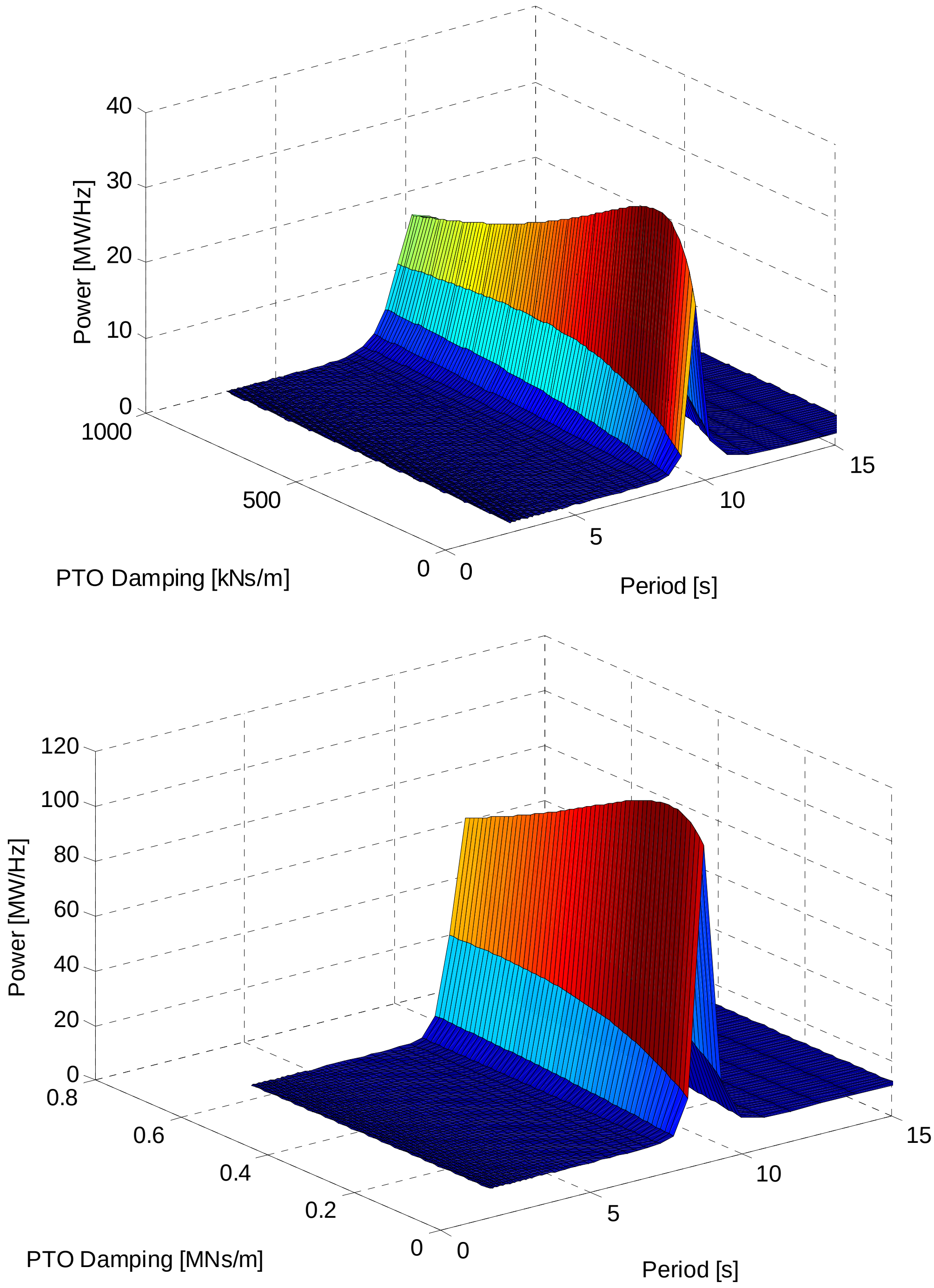

Figure 4: Power absorption characteristics of a single heaving cylinder (as function of the PTO damping coefficient and incident wave period in a irregular sea state): Case Id. 4 (top) and Case Id. 6 (bottom), following Table 1

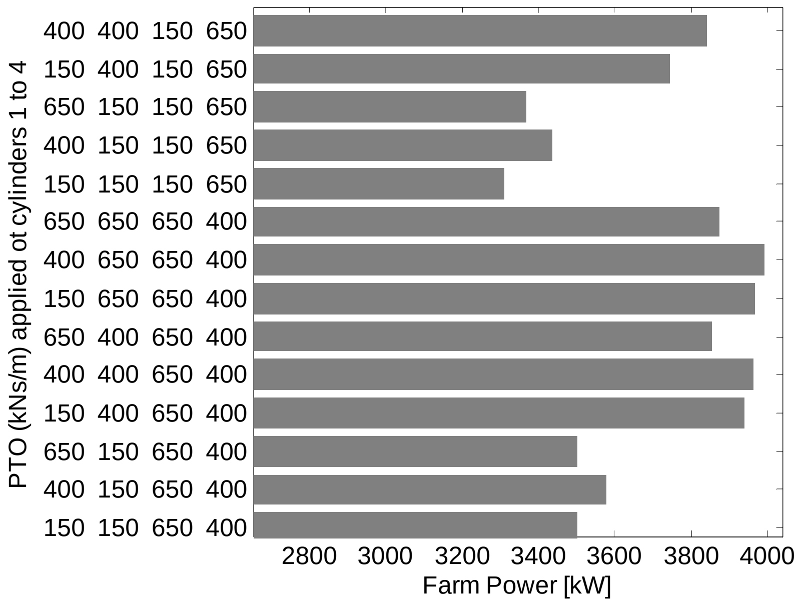

Figure 5: Wave farm power absorption characteristics as function of the PTO damping applied to cylinders 1 to 4 (left to right on the vertical axis label);
WEC spacing = 6a; $\beta = -\pi / 4$ rad;
Wave conditions: JONSWAP, H_{m0} = 4m; T_{peak} = 10.2s